The problem of inverted elements when simulating deformable objects

Toni Susín

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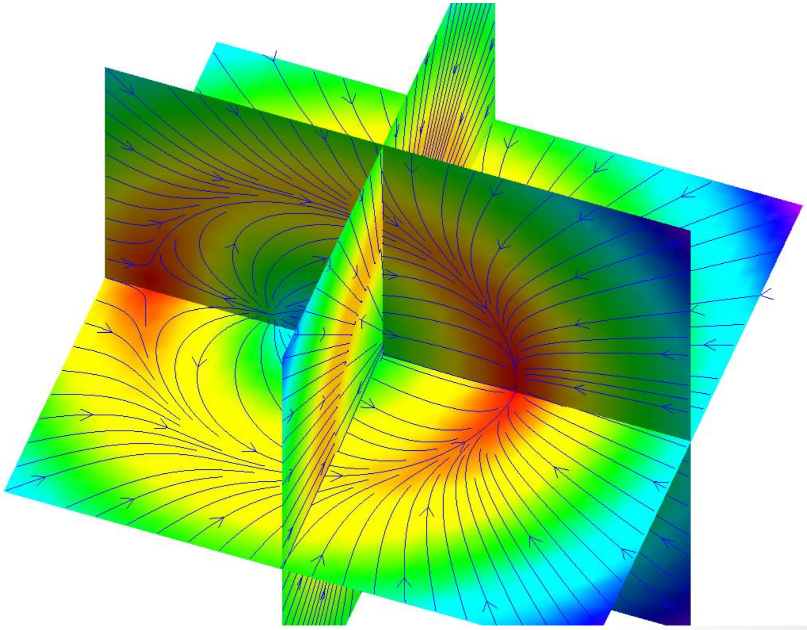
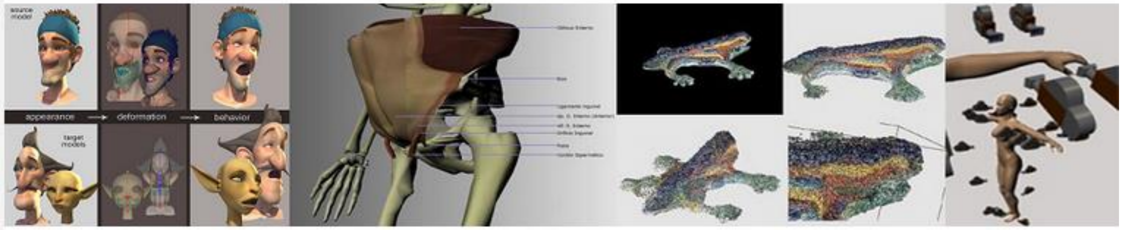


Toni Susin

Last modified 19/06/2018



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Short CV

Toni Susin is an Associate Professor of Applied Mathematics at UPC-BarcelonaTech . He is the head of the **Dynamic Simulation Lab** included in the ViRVIG research group in Barcelona. His major expertise is in numerical methods and physically-based simulation applied to images and computer graphics.

He has been a visiting professor at the Univ. of Minnesota, Minneapolis (1992), the Univ. Autonoma Metropolitana, Mexico DF. (1995), the Univ. of California, Irvine (2002), the Univ. of Zürich, Zürich (2010), the Addis Ababa Univ., Ethiopia (2012), Seoul National University (2017). He has served in the Program Committee member/reviewer of several international conferences and workshops.

He also has been co-founder of three different technological companies during the last years.

NUMERICAL FACTORY our present teaching project.

Permanent address

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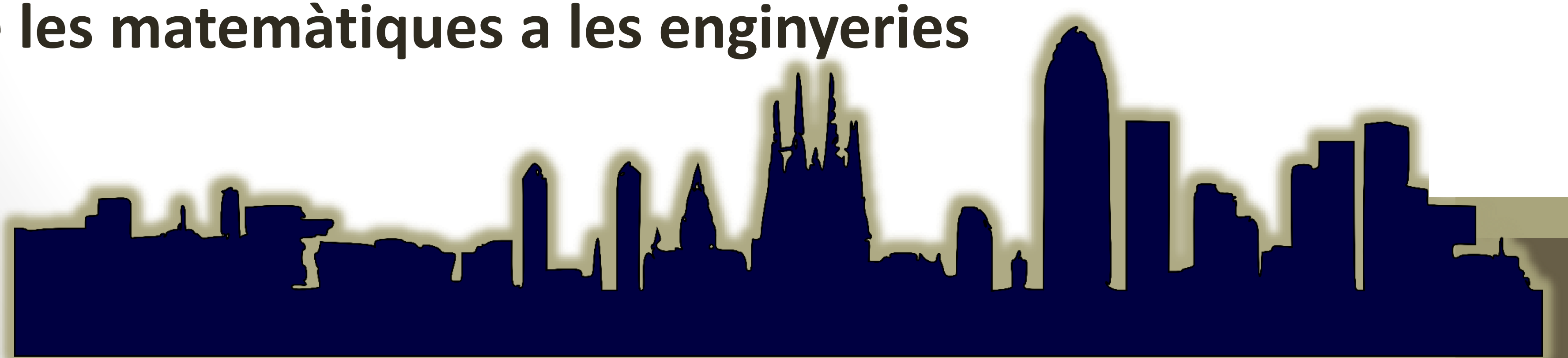
  Diagonal 647, E-08028 Barcelona, Spain
- **Tel:** +34 934017781

Numerical Factory:

by Toni Susín

Un tast numèric sobre l'ensenyament de les matemàtiques a les enginyeries



Physically-Based Animation

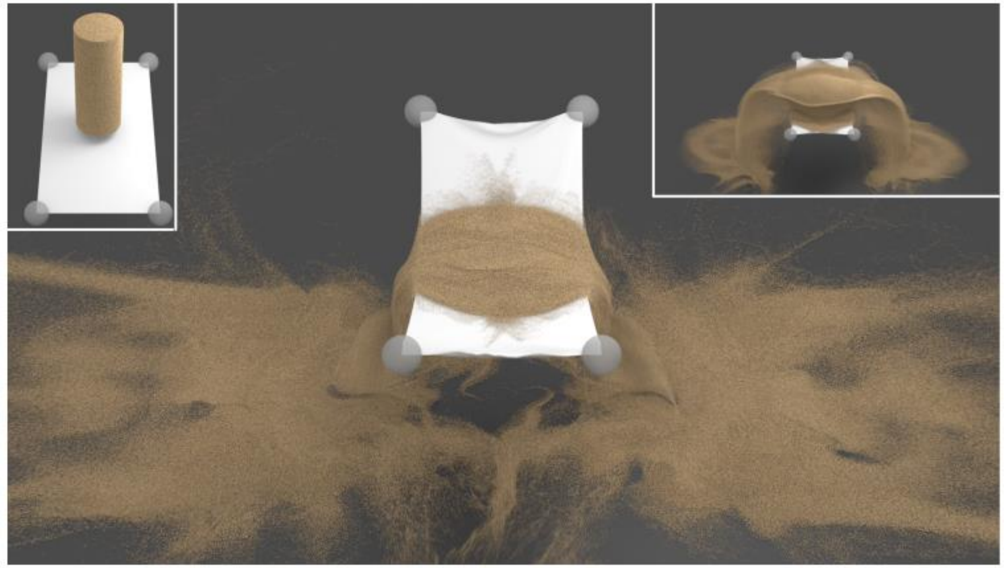
- **Main Tools:**
  - Programming
  - Numerical Integration of Differential Equations
  - Computer Graphics
  - 3D modeling

Physically-Based Animation

- **Main Tools:**
  - Programming
  - Numerical Integration of Differential Equations
  - Computer Graphics
  - 3D modeling

- **Applications:**
  - Special Effects (FX) for movies
  - FX for TV commercials
  - Video Games

Movie Animation: A Continuum Approach for Frictional Contact

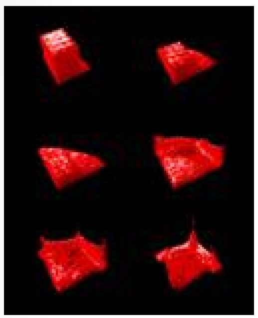


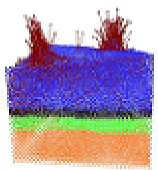
Joseph Teran

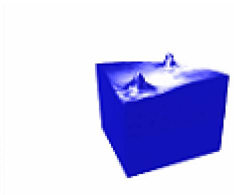
http://www.math.ucla.edu/~jteran/

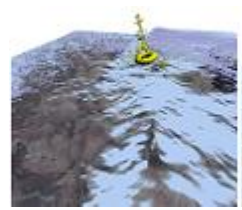
Physically-Based Animation

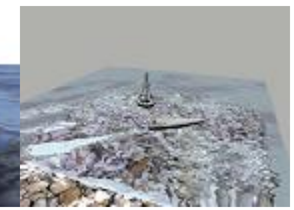
Fluid Simulation: (N. Suarez-2007, J. Ojeda-2013)











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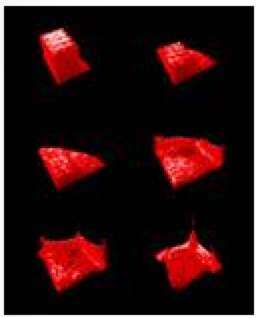
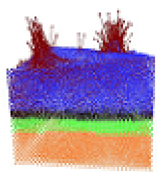
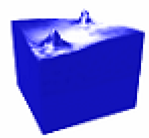
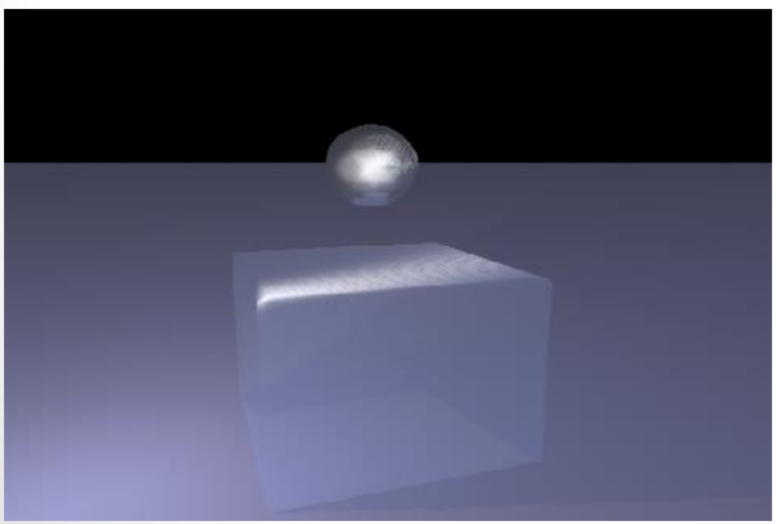
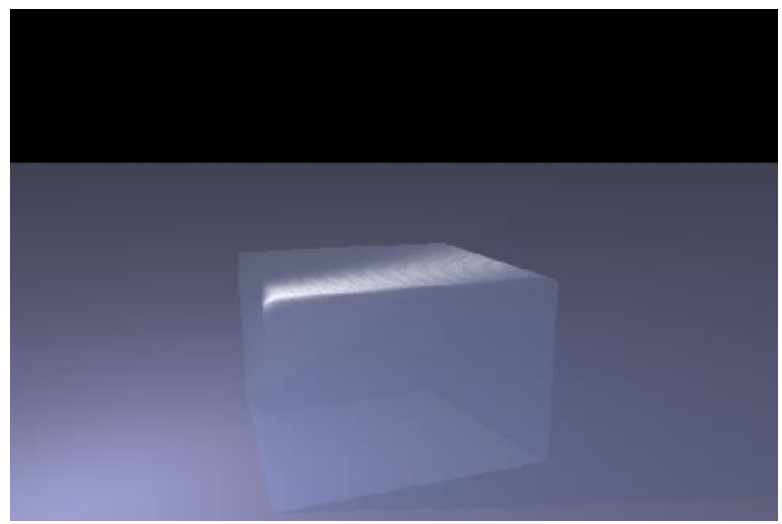
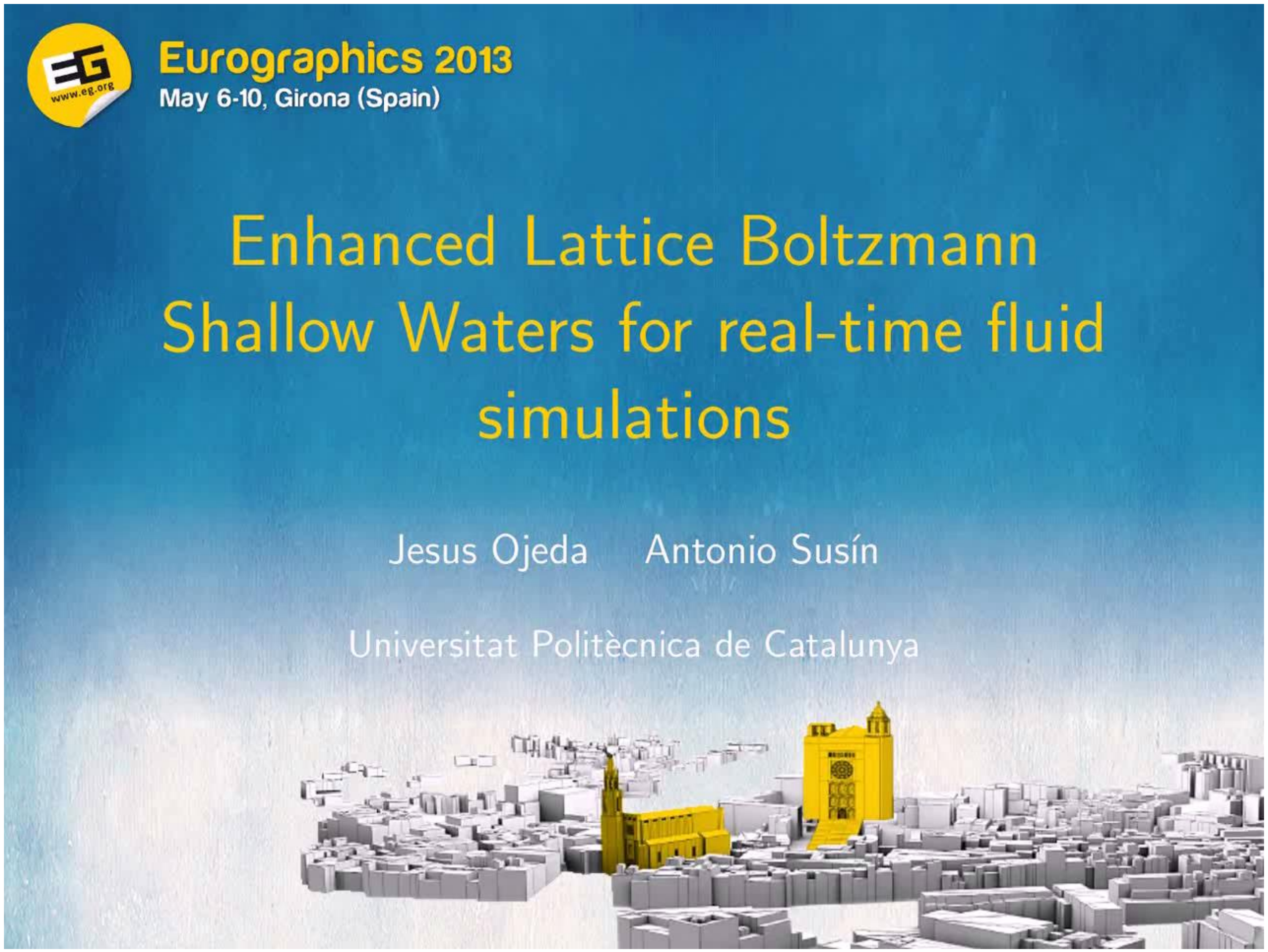




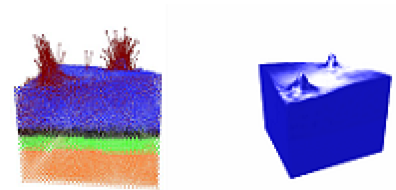



Physically-Based Animation

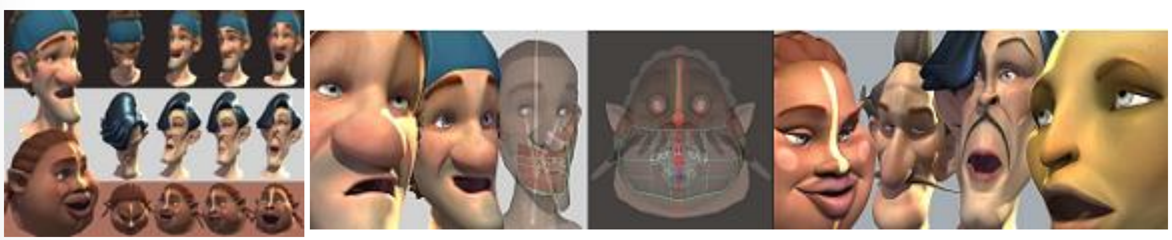


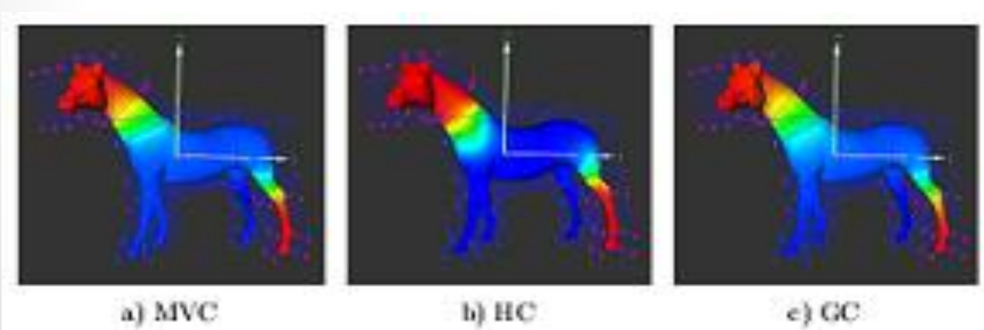
Physically-Based Animation

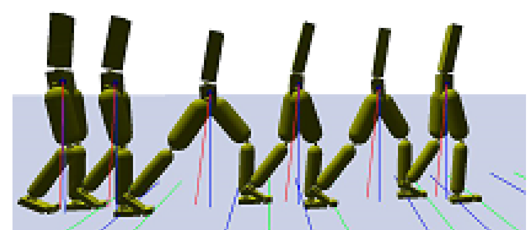
Fluid Simulations: (J. Ojeda-2013, N. Suarez-2007)



Animation: (V. Costa-Orvalho 2007, J. Rodríguez 2008, J. R. Nieto 2013)

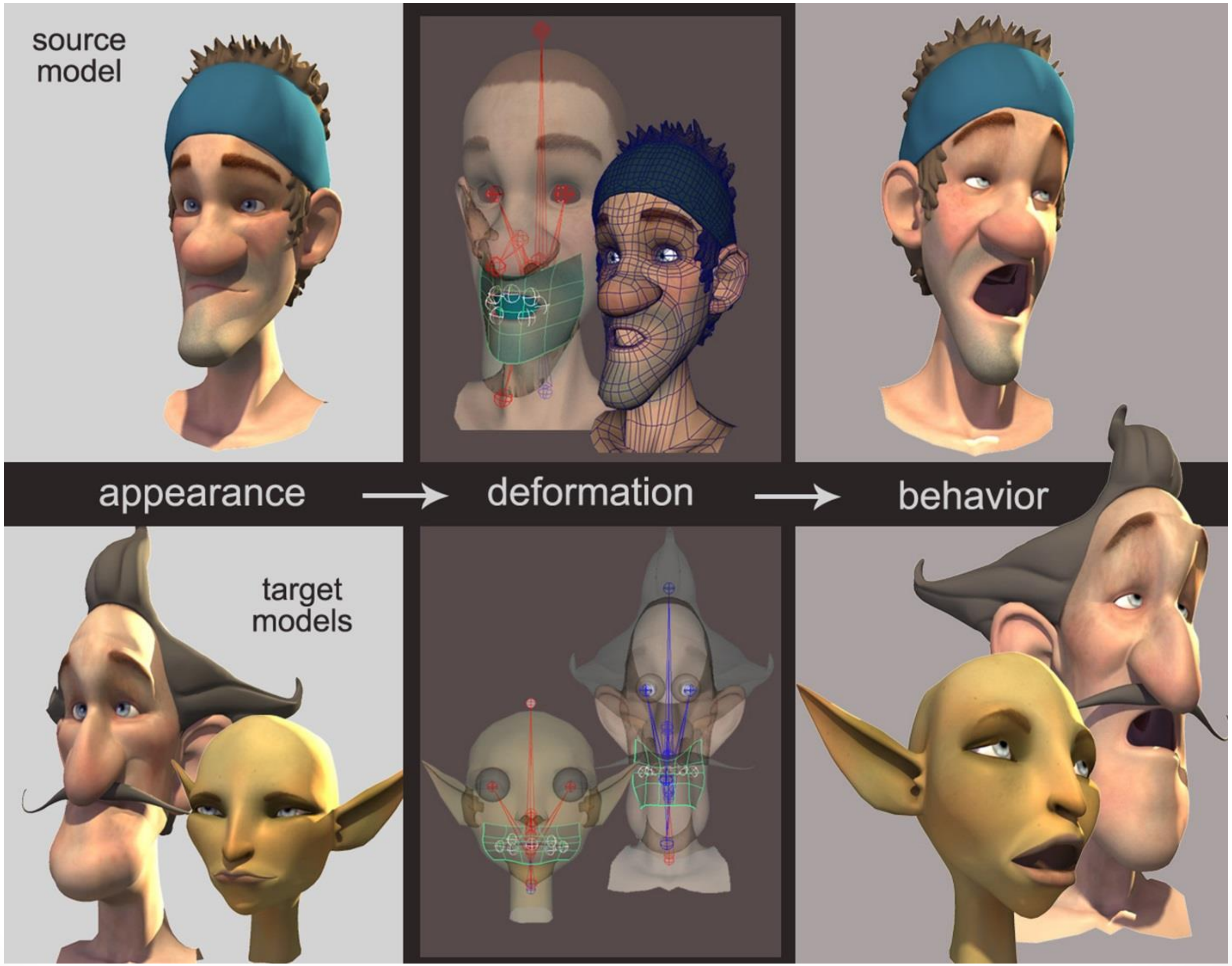






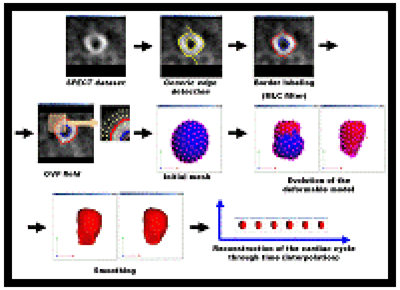
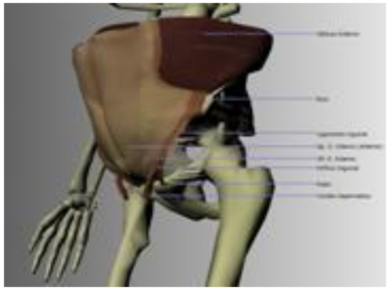
Character Animation

Retargeting Facial Animation: (V. Costa-Orvalho, A. Susín 2007)



Physically-Based Animation

Medicine: (O. García-2004, G. Fortuny-2009, J. Roca-2010)

Physically-Based Animation

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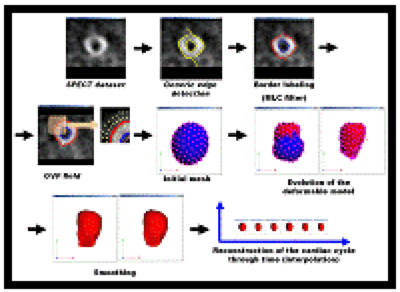
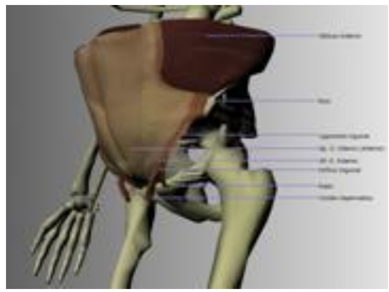
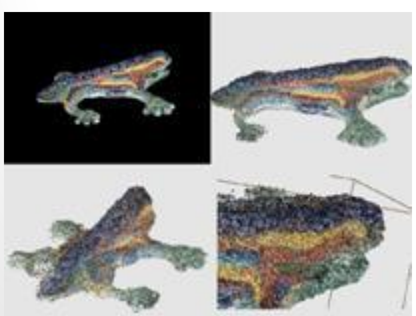
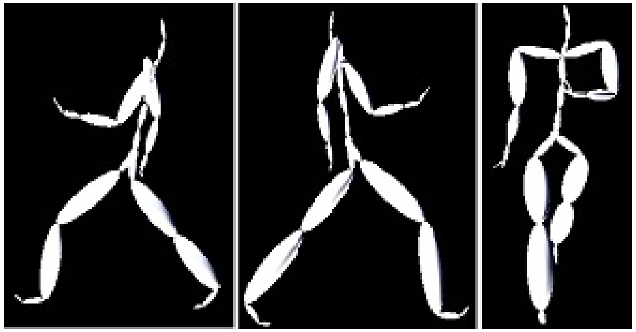



Image-Based Modeling: (M. Sainz 2003)



Physically-Based Animation

Motion Capture (A. Baena 2013 – S. Mutlu 2010)

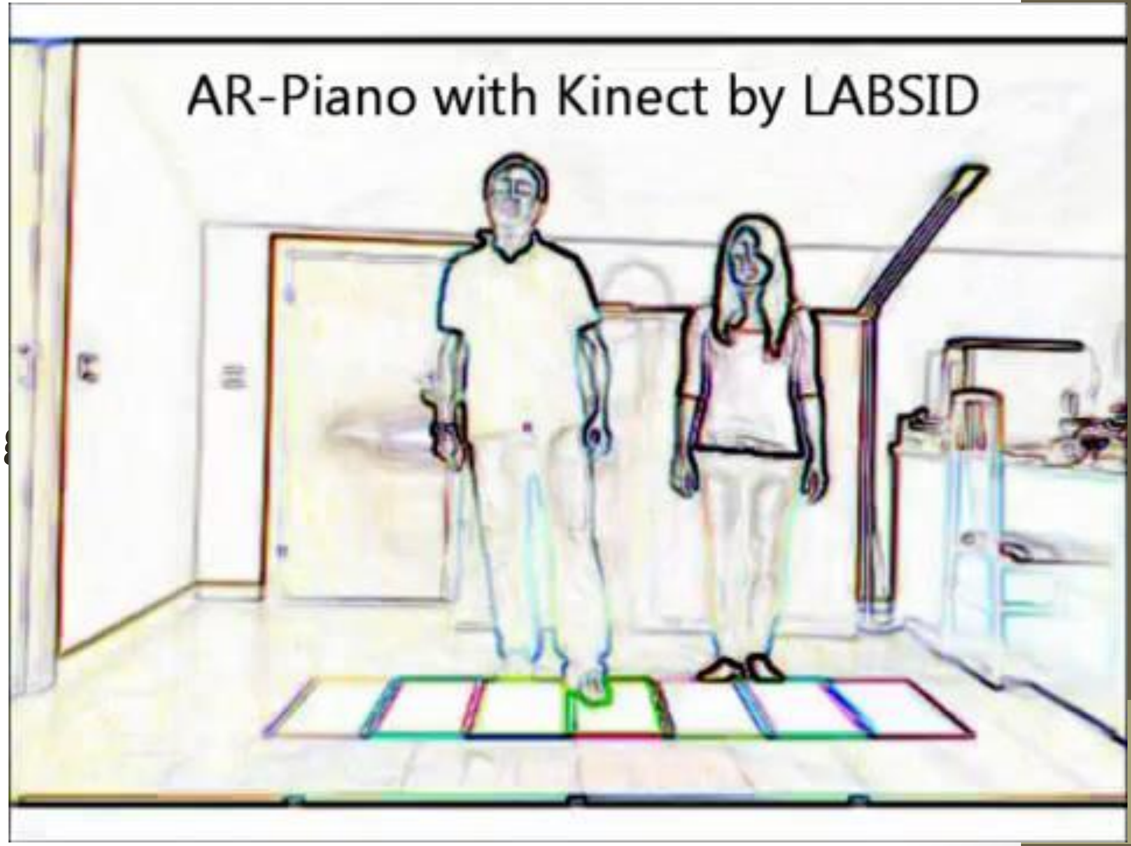


Augmented Reality (K. Anglès-2011)

Physically-Based Animation

Motion Capture (A. Baena 2013 – S. Mutlu 2010)

Augmented Reality (K. An



The problem of inverted elements when simulating deformable objects

Oscar Civit (Rockstar Games)

Toni Susín (UPC-BcnTech)

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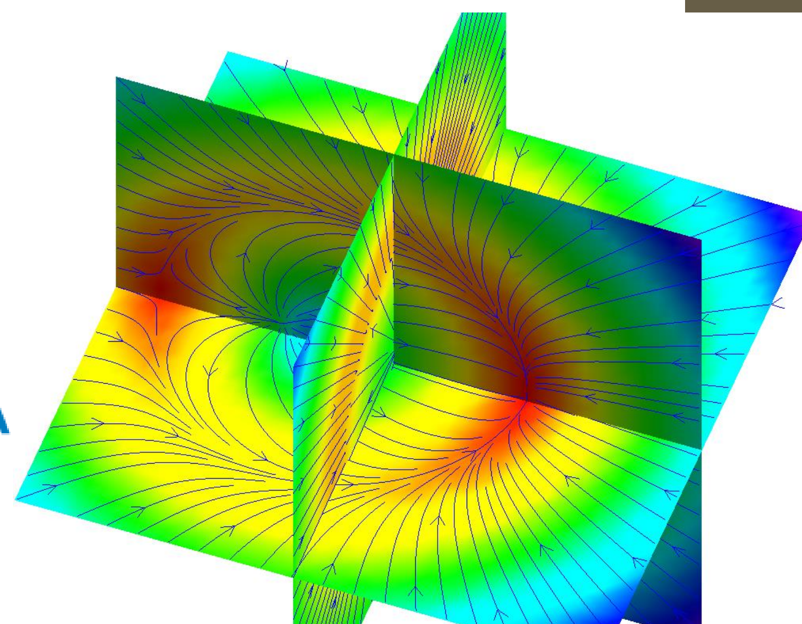

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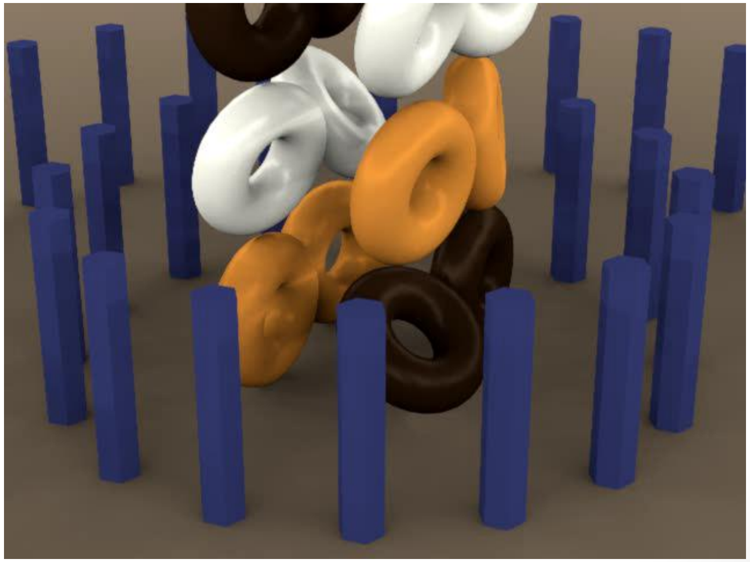
- Deformable objects
- Finite Element Method (FEM)
  - Linear FEM
  - Co-Rotational FEM
- Inverted Elements
  - Forces on inverted elements
  - Robust Time-coherent solution

Deformable Objects

- Rigid Bodies

- Deformable Bodies



Deformable Objects

1d: Ropes, hair

Deformable Objects

1d: Ropes, hair

2d: Cloth, clothing

Deformable Objects

1d: Ropes, hair

2d: Cloth, clothing

3d: Fat, tires, organs

Continuum Mechanics

- **Deformation map:** $\mathbf{P}\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3, \quad \mathbf{p} = \mathbf{P}(\mathrm{x})$

Magnitudes:

- **Displacements**: Motion of each point

$$\mathbf{u}(\mathrm{x}) = \mathbf{P}(\mathrm{x}) - \mathrm{x}$$

- **Strain**: Relative elongation (or compression) of the material.

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\mathrm{x})$$

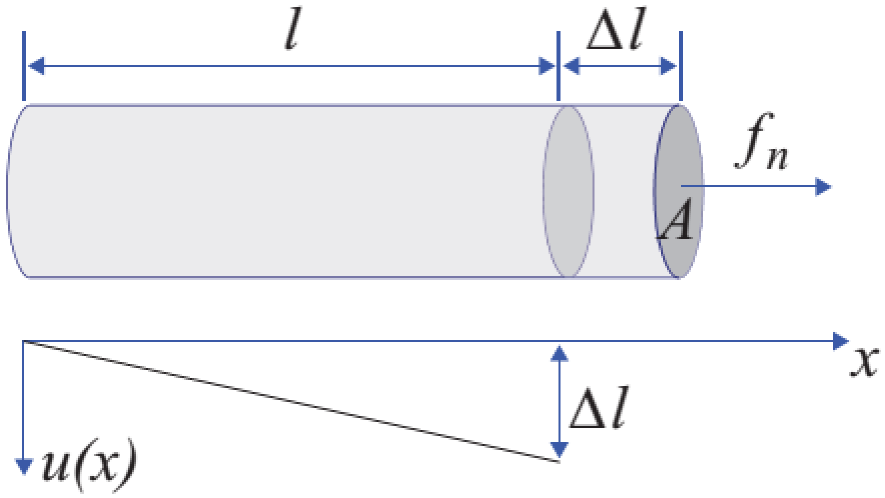
- **Stress**: Force per unit area.

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathrm{x})$$

Continuum Mechanics

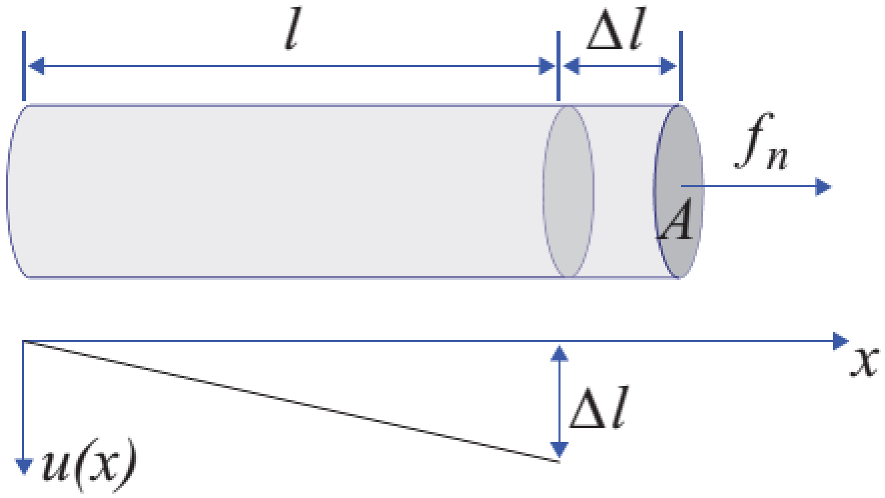
- **Constitutive Laws: Stress - Strain**

Hooke's law



Continuum Mechanics

- **Constitutive Laws:** **Stress - Strain**

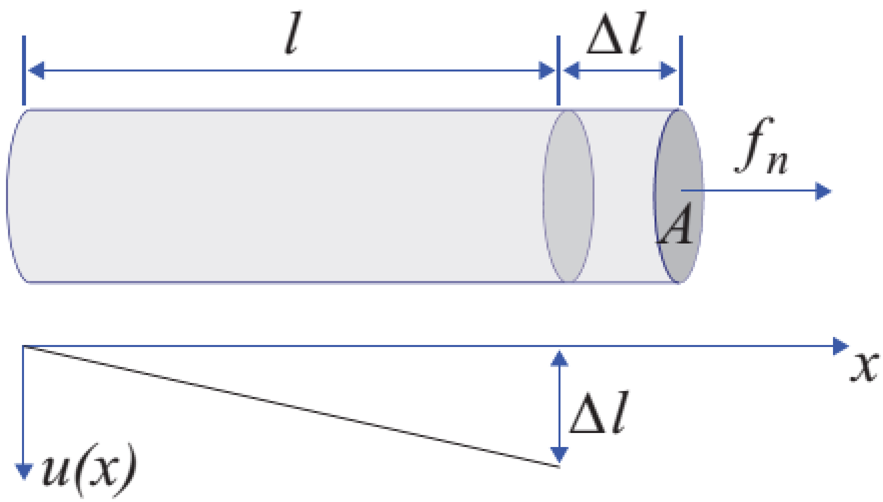


Hooke's law

$$\frac{f_n}{A} = E\frac{\Delta l}{l}.$$

Continuum Mechanics

- **Constitutive Laws: Stress - Strain**



Hooke's law

$$\frac{f_n}{A} = E\frac{\Delta l}{l}.$$

$$\sigma = E\varepsilon,$$

Stress **Strain**

Young's modulus.

Continuum Mechanics

$$\sigma = E\varepsilon,$$

Stress **Strain**

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

Continuum Mechanics

$\sigma = E\varepsilon,$

Stress **Strain**

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \qquad \varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

- For linear Isotropic materials:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix}$$

E is Young's modulus

$\nu \in [0 \ldots \frac{1}{2})$ Poisson's ratio

Continuum Mechanics

- **Constitutive Laws:** **Strain - Displacements**

Strain Tensor:

$$\boldsymbol{\varepsilon}_{\mathrm{G}} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}} + \nabla\mathbf{u}^{\mathrm{T}} \cdot \nabla\mathbf{u})$$

Green's nonlinear strain tensor

$$\boldsymbol{\varepsilon}_{\mathrm{G}} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}})$$

Cauchy's linear strain tensor

Continuum Mechanics

- **Dynamical model:**

$$\rho \ddot{\boldsymbol{u}} = \mathbf{f}_{\text{elast}} + \mathbf{f}_{\text{ext}}$$

$$\rho \ddot{\boldsymbol{u}} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f}_{\text{ext}}$$

Second order hyperbolic PDE

Numerical solution:

- Finite Element Method
- Implicit Euler Method

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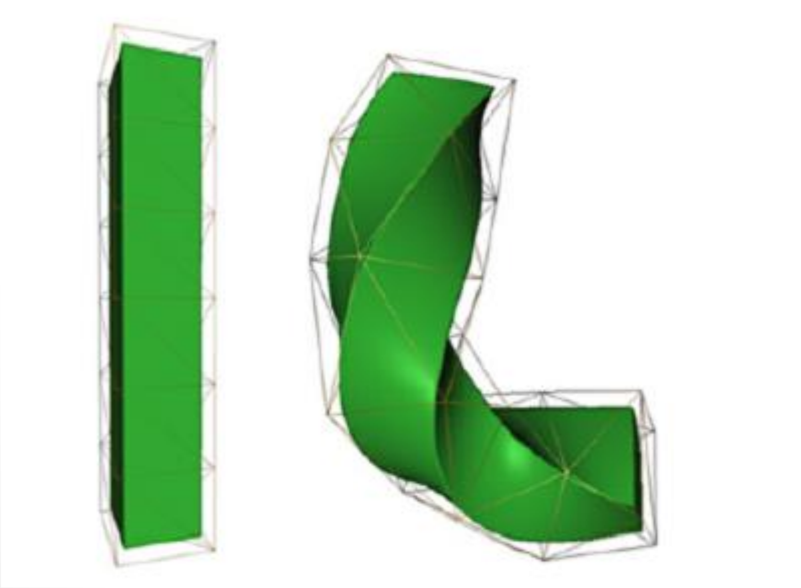
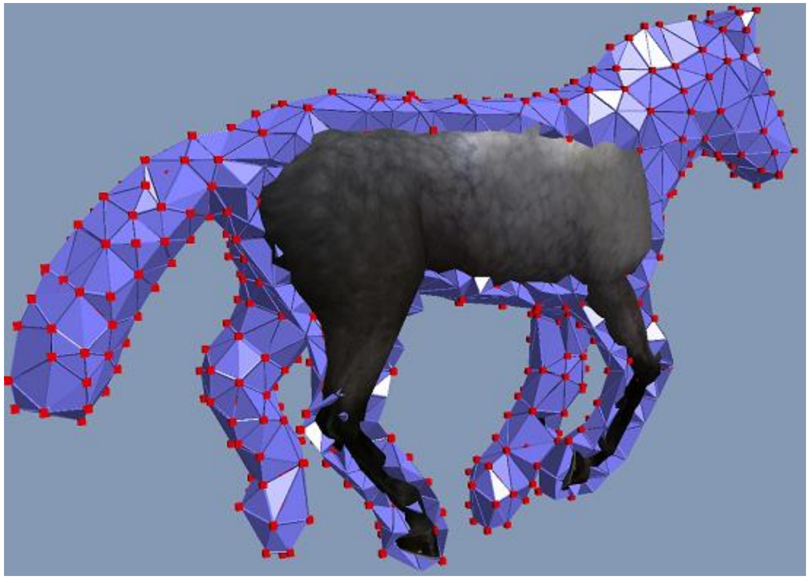
Finite Element Method

- Represent the geometry of the model by a set of finite elements.



[Jin Huang et al, 2009]

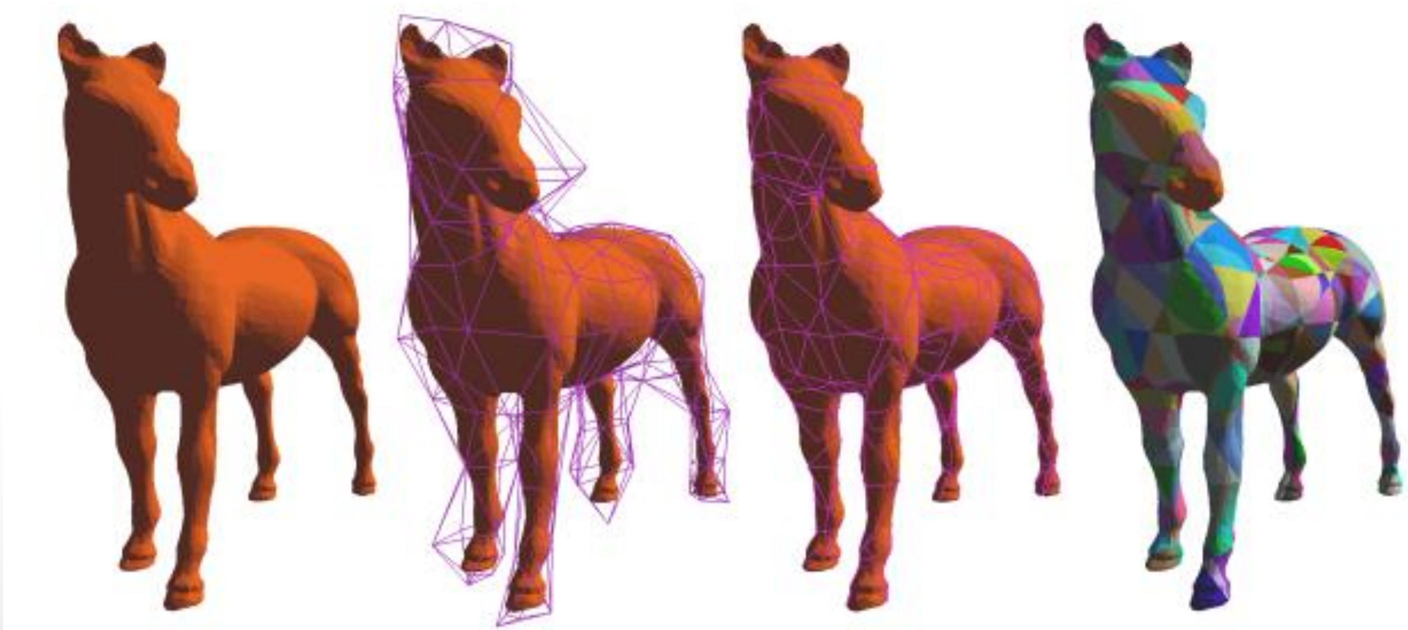


[Müller et al., 2008]

Finite Element Method

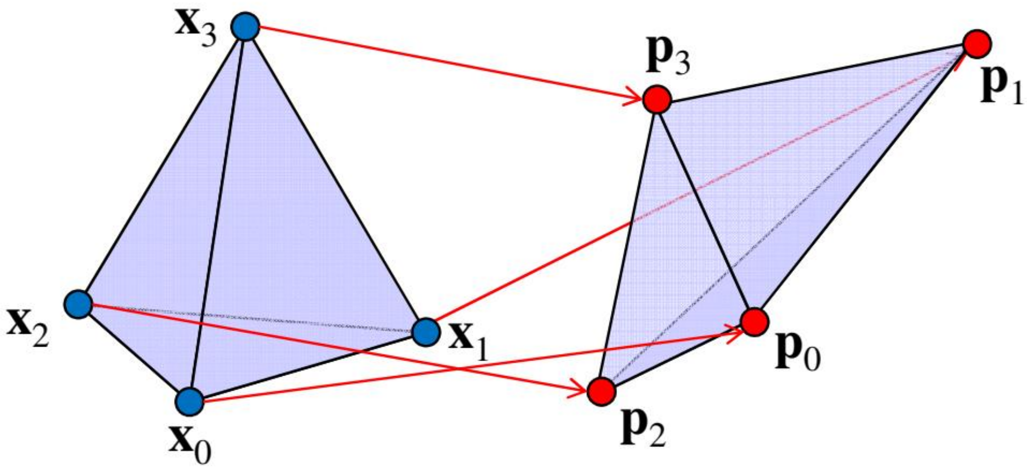
- Interactive simulation of deformable solids using Finite Element Methods (FEM).
- Use a coarse mesh for simulation.

[O.Civit-F, A.Susin, 2016]



Linear FEM

- **Linear Tetrahedral elements**:



Using *barycentric coordinates*, $\boldsymbol{\alpha}$

$$\mathbf{x} = [\mathbf{x}_1 - \mathbf{x}_0 \quad \mathbf{x}_2 - \mathbf{x}_0 \quad \mathbf{x}_3 - \mathbf{x}_0] \cdot \boldsymbol{\alpha}$$
$$\mathbf{p} = [\mathbf{p}_1 - \mathbf{p}_0 \quad \mathbf{p}_2 - \mathbf{p}_0 \quad \mathbf{p}_3 - \mathbf{p}_0] \cdot \boldsymbol{\alpha}$$

and then

$$\mathbf{p} = [\mathbf{p}_1 - \mathbf{p}_0 \quad \mathbf{p}_2 - \mathbf{p}_0 \quad \mathbf{p}_3 - \mathbf{p}_0] \cdot [\mathbf{x}_1 - \mathbf{x}_0 \quad \mathbf{x}_2 - \mathbf{x}_0 \quad \mathbf{x}_3 - \mathbf{x}_0]^{-1} \cdot \mathbf{x}$$

Linear FEM

<u>Deformation map for each element:</u> (3x3 matrix)

$$\mathbf{P} = [\mathbf{p}_1 - \mathbf{p}_0 \quad \mathbf{p}_2 - \mathbf{p}_0 \quad \mathbf{p}_3 - \mathbf{p}_0] \cdot \underbrace{[\mathbf{x}_1 - \mathbf{x}_0 \quad \mathbf{x}_2 - \mathbf{x}_0 \quad \mathbf{x}_3 - \mathbf{x}_0]^{-1}}_{\text{constant}}$$

$\mathbf{p} = \mathbf{P} \cdot \mathbf{x}$ linear map

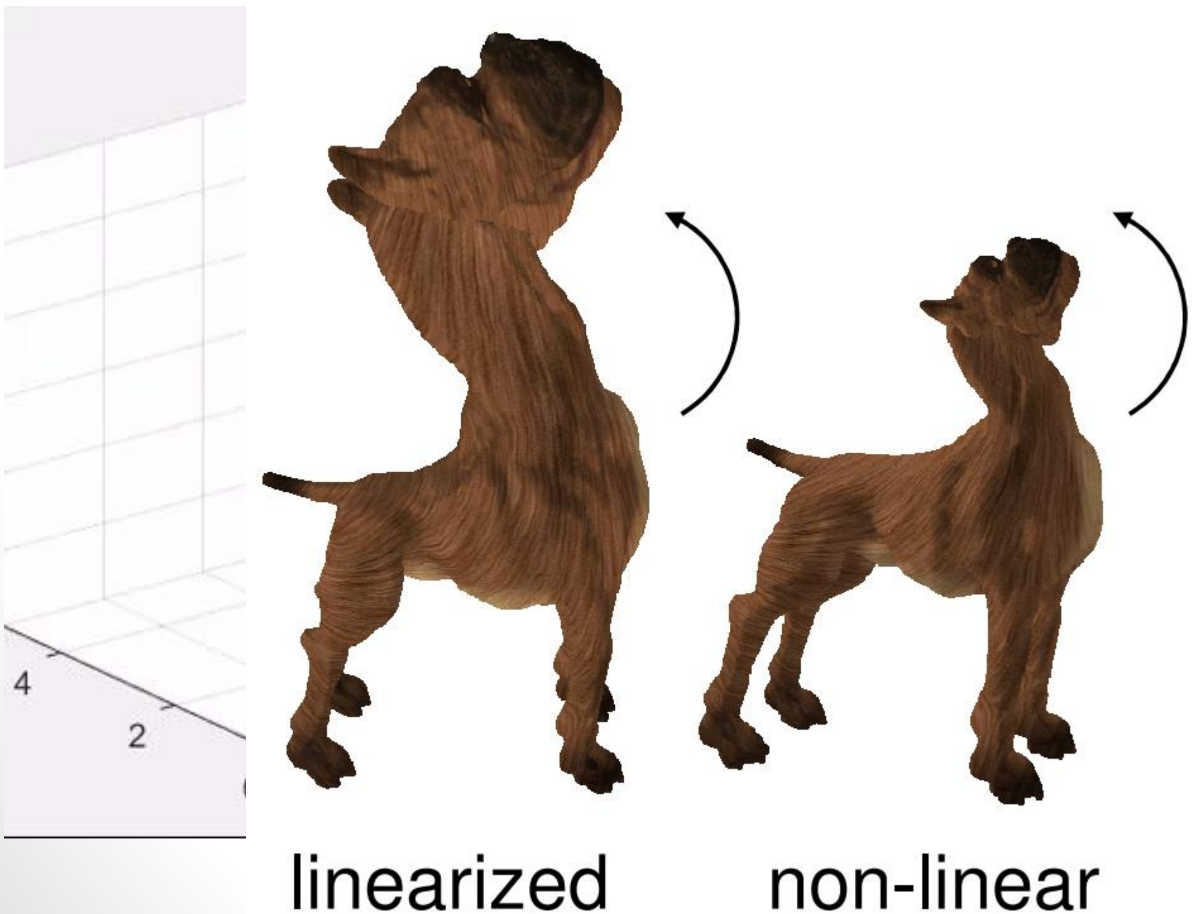
$\mathbf{u} = \mathbf{P} \cdot \mathbf{x} - \mathbf{x}, \quad \nabla\mathbf{u} = \mathbf{P} - \mathbf{I},$ Displacements

$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}})$ Strain (Cauchy)

$\boldsymbol{\sigma} = \mathrm{E}\boldsymbol{\varepsilon}$ Stress

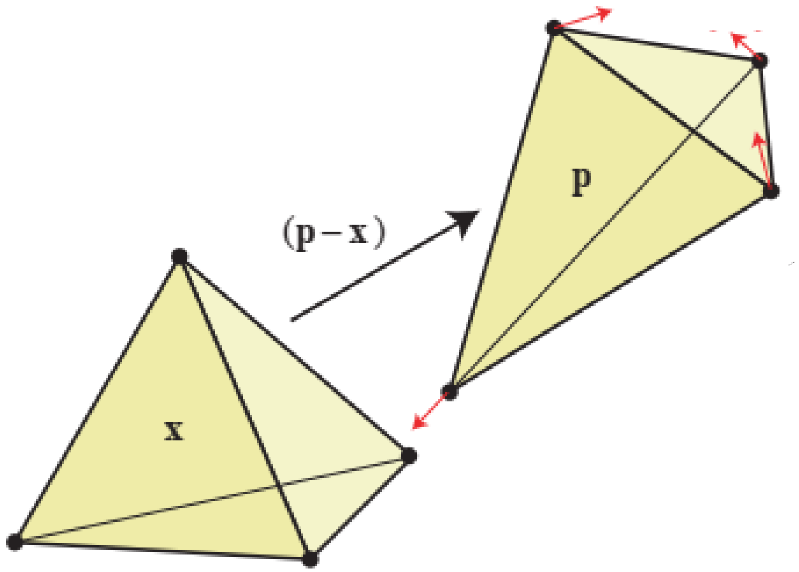
Linear FEM

Fast but only appropriate for small deformations. Increase volume under large rotational deformations.



Co-rotational FEM

Non-Linear approach consisting on decoupling rotation from the deformation map in order to correct the increasing volume problem.



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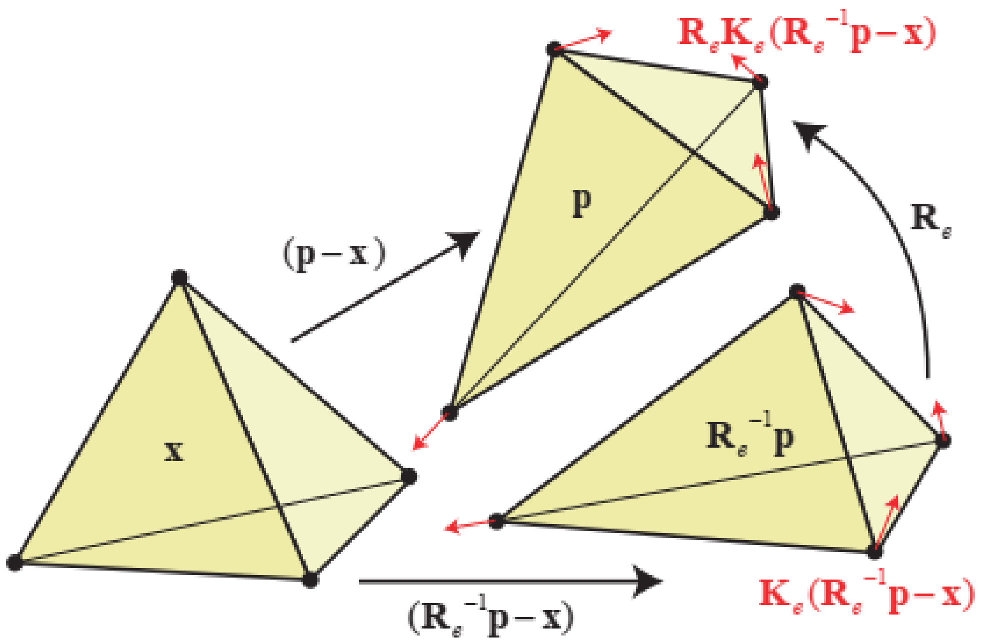
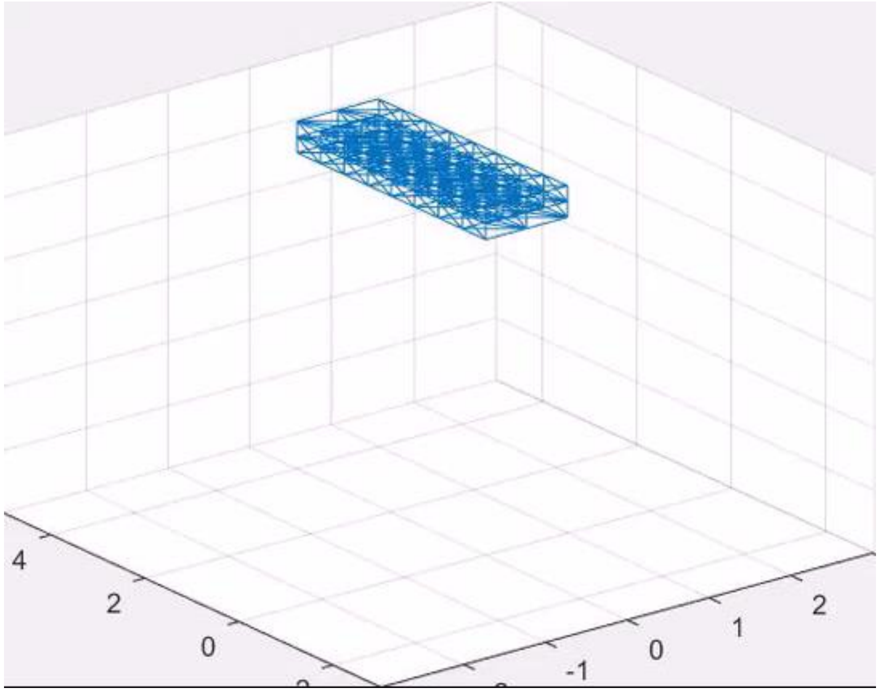


Co-rotational FEM

Decouple rotation from the deformation map in order to correct the increasing volume problem.



Decomposition methods

- Transformation Matrix

$$\mathbf{P} = [\mathbf{p}_1 - \mathbf{p}_0, \mathbf{p}_2 - \mathbf{p}_0, \mathbf{p}_3 - \mathbf{p}_0][\mathbf{x}_1 - \mathbf{x}_0, \mathbf{x}_2 - \mathbf{x}_0, \mathbf{x}_3 - \mathbf{x}_0]^{-1}$$

Decomposition methods

- Transformation Matrix

$$\mathbf{P} = [\mathbf{p}_1 - \mathbf{p}_0, \mathbf{p}_2 - \mathbf{p}_0, \mathbf{p}_3 - \mathbf{p}_0][\mathbf{x}_1 - \mathbf{x}_0, \mathbf{x}_2 - \mathbf{x}_0, \mathbf{x}_3 - \mathbf{x}_0]^{-1}$$

- Numerical Decomposition Methods:
  - **QR:** $\mathbf{P} = \mathrm{RS}$

    R orthogonal: Rotation Matrix

    (Gram-Schmidt orthogonalization)

Decomposition methods

- Transformation Matrix

$$\mathbf{P} = [\mathbf{p}_1 - \mathbf{p}_0, \mathbf{p}_2 - \mathbf{p}_0, \mathbf{p}_3 - \mathbf{p}_0][\mathbf{x}_1 - \mathbf{x}_0, \mathbf{x}_2 - \mathbf{x}_0, \mathbf{x}_3 - \mathbf{x}_0]^{-1}$$

- Numerical Decomposition Methods:
  - **QR:**
  - **SVD:** $\mathbf{P} = \mathrm{UDV}^{\mathrm{T}}$

    UV^{T} orthogonal ≡ Rotation Matrix

Decomposition methods

- Transformation Matrix

$$\mathbf{P} = [\mathbf{p}_1 - \mathbf{p}_0, \mathbf{p}_2 - \mathbf{p}_0, \mathbf{p}_3 - \mathbf{p}_0][\mathbf{x}_1 - \mathbf{x}_0, \mathbf{x}_2 - \mathbf{x}_0, \mathbf{x}_3 - \mathbf{x}_0]^{-1}$$

- Numerical Decomposition Methods:
  - **QR:**
  - **SVD:**
  - **Polar Decomposition:**

$$\mathrm{R}_0 = \mathbf{P}, \qquad \mathrm{R}_{k+1} = \frac{1}{2}(\mathrm{R}_k + \mathrm{R}_k^{-T})$$

$\mathrm{R}_{k+1} \approx \mathrm{R}_k \equiv$ Rotation Matrix

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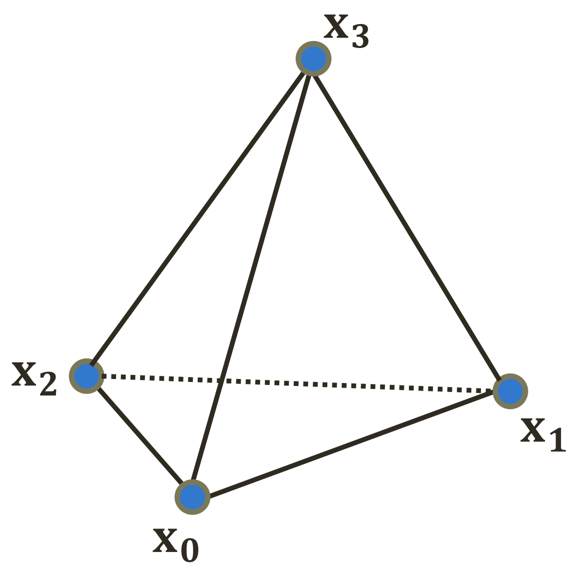
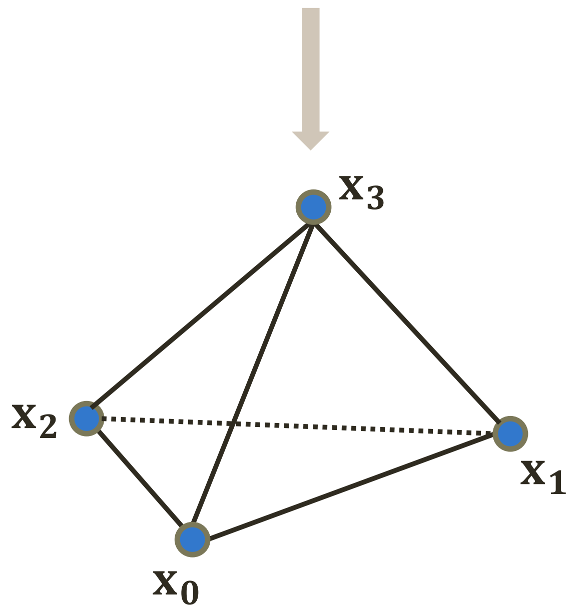
Inverted Elements

Element degeneration threatens robustness and realism:



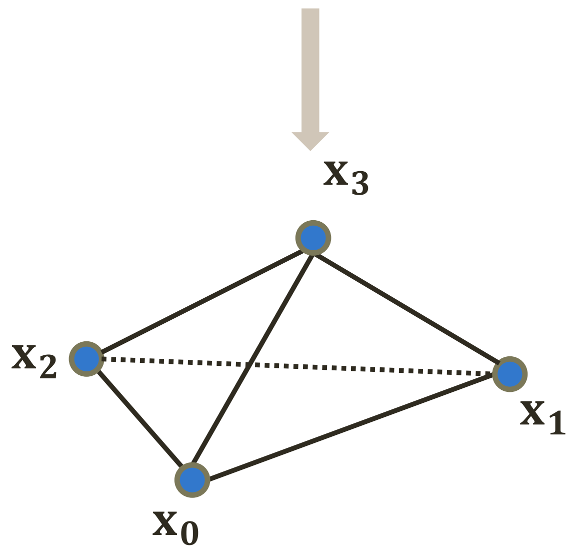
Inverted Elements

Element degeneration threatens robustness and realism:



Inverted Elements

Element degeneration threatens robustness and realism:



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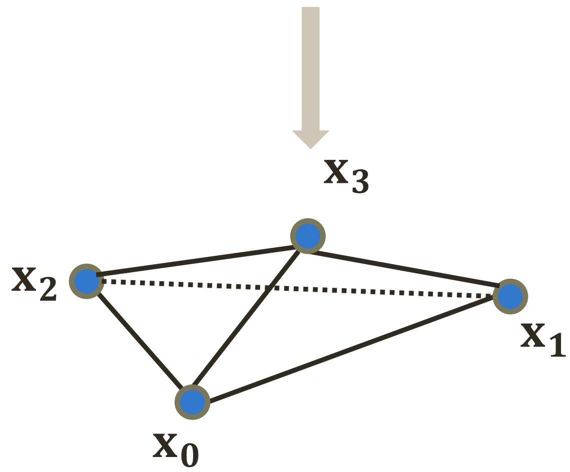

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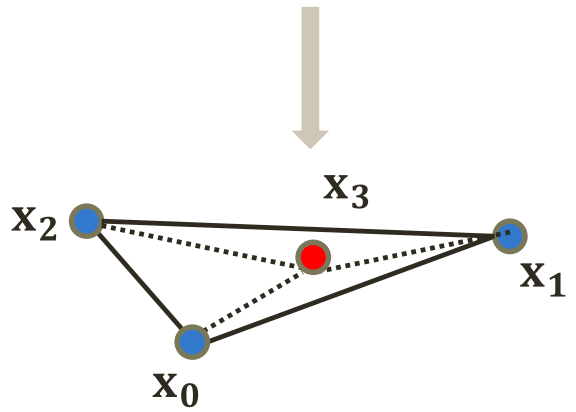

Collapse!!

Inverted Elements

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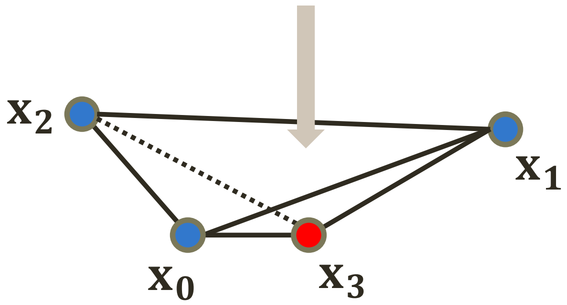

Degenerate!!

Inverted Elements

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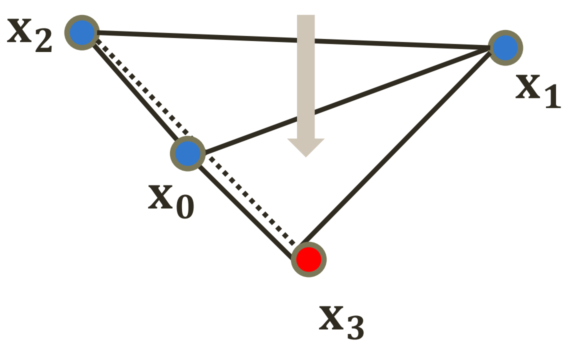
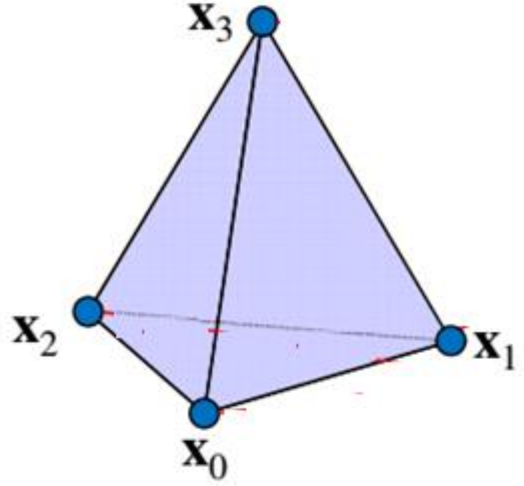


Degenerate!!

Inverted Elements



Element Volume

$$\mathrm{V} = \frac{1}{6}\det(\mathbf{x_3} - \mathbf{x_0}, \mathbf{x_2} - \mathbf{x_0}, \mathbf{x_1} - \mathbf{x_0})$$

$\mathrm{V} \approx 0$ **Collapse configuration**

$\mathrm{V} < 0$ **Inverted configuration**

Inverted Elements

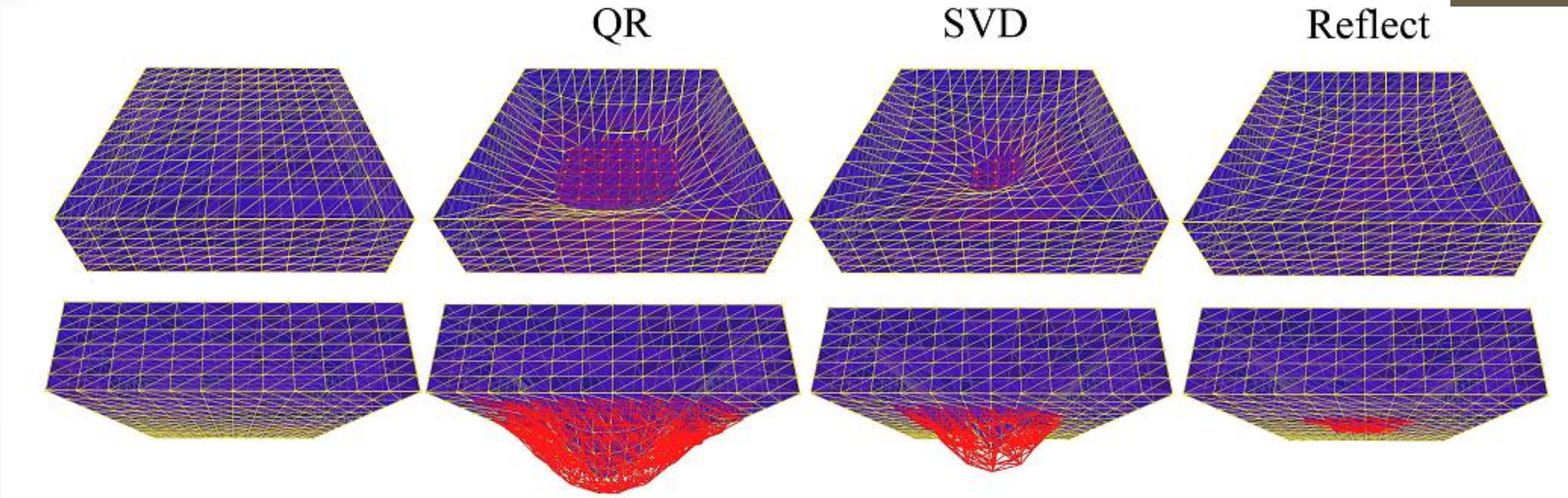
Element degeneration threatens robustness and realism:

- We identify **issues** with existing degenerate element treatment schemes

Contribution

Element degeneration threatens robustness and realism:

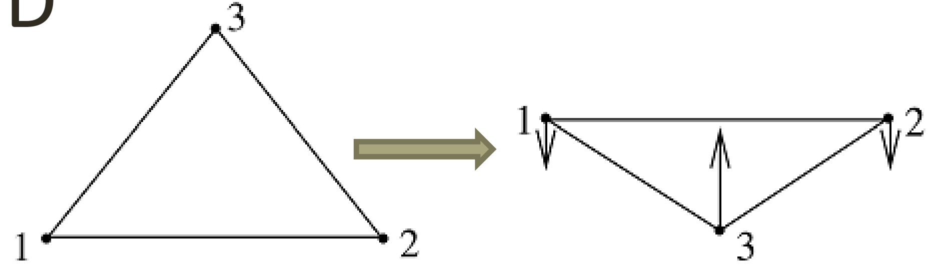
- We identify **issues** with existing degenerate element treatment schemes
- We propose a **new method** that avoids them



Forces on Inverted Elements

Corotational Force 2D

$$\mathbf{f} = \mathbf{RK}(\mathbf{R}^{\mathbf{T}}\mathbf{x} - \mathbf{p})$$



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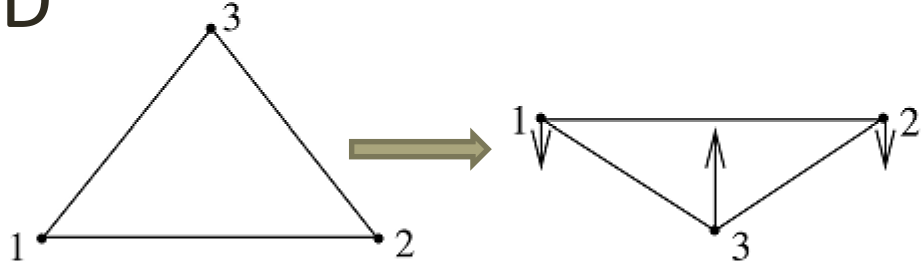
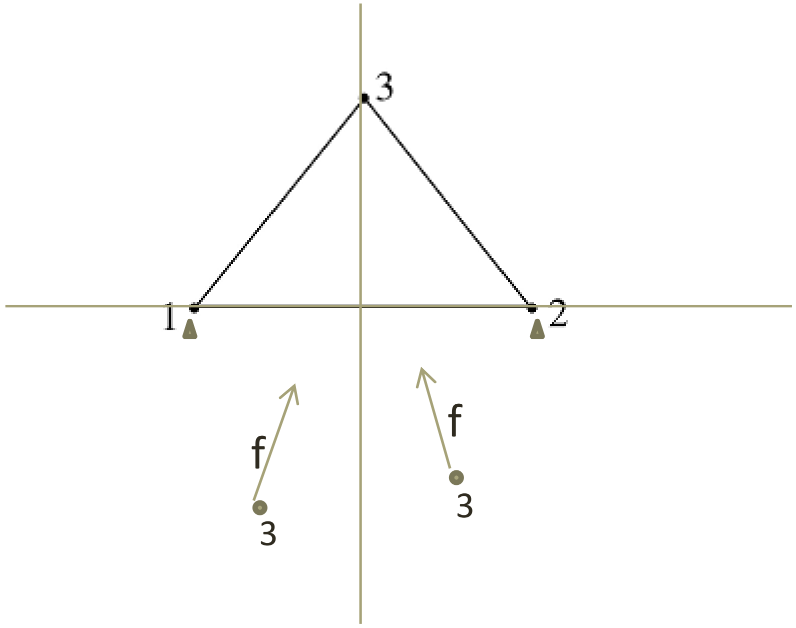
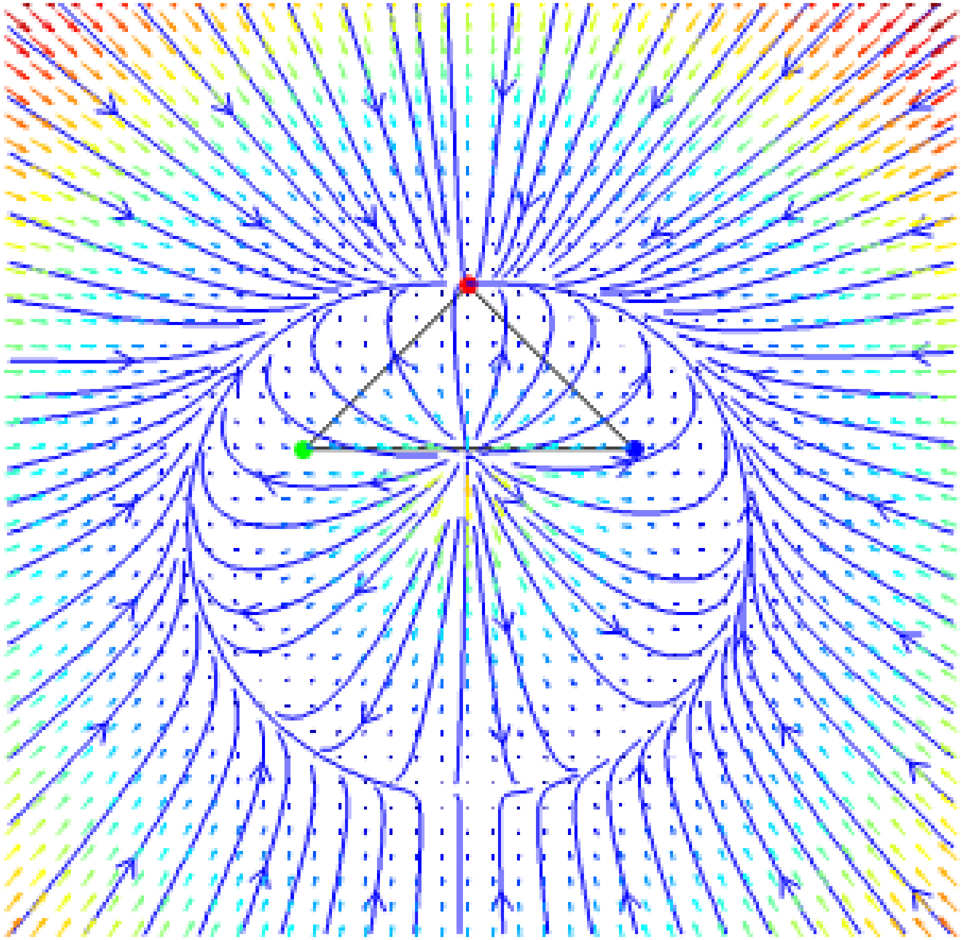




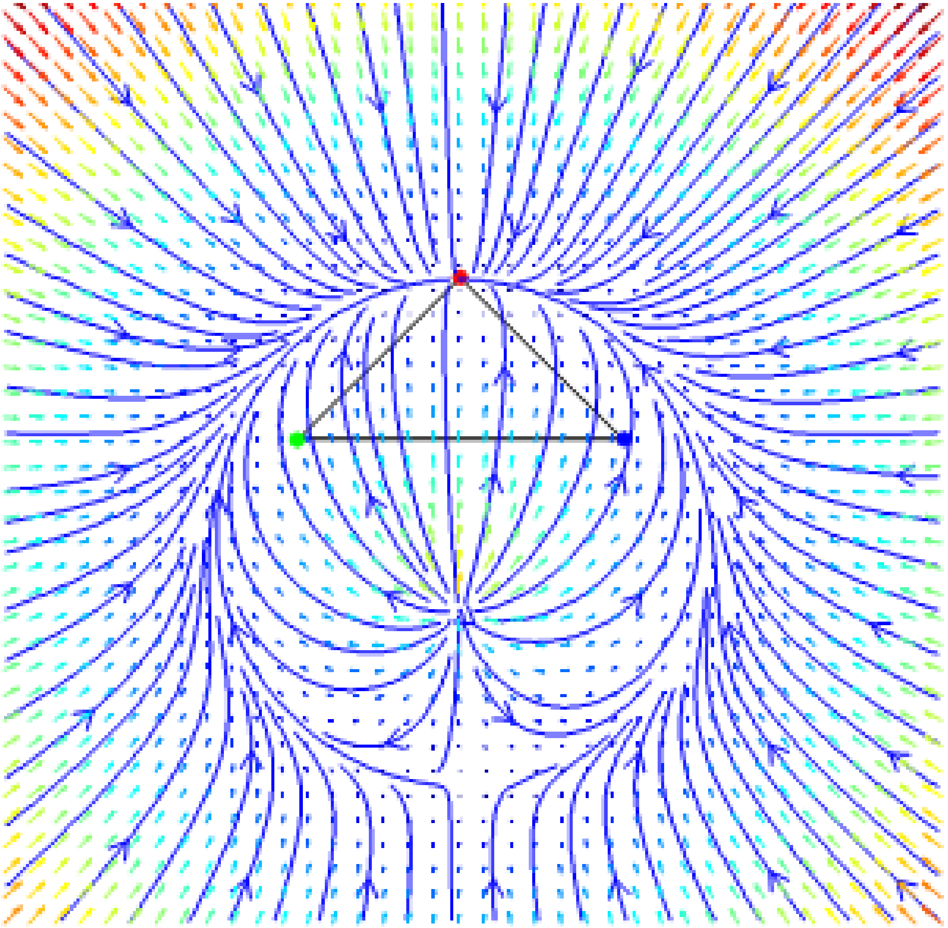
Forces on Inverted Elements

QR



Forces on Inverted Elements

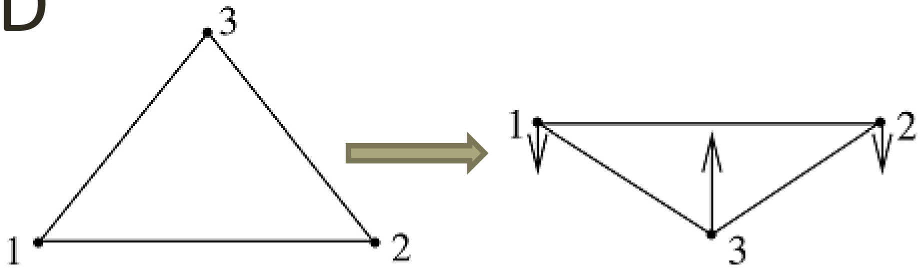
SVD



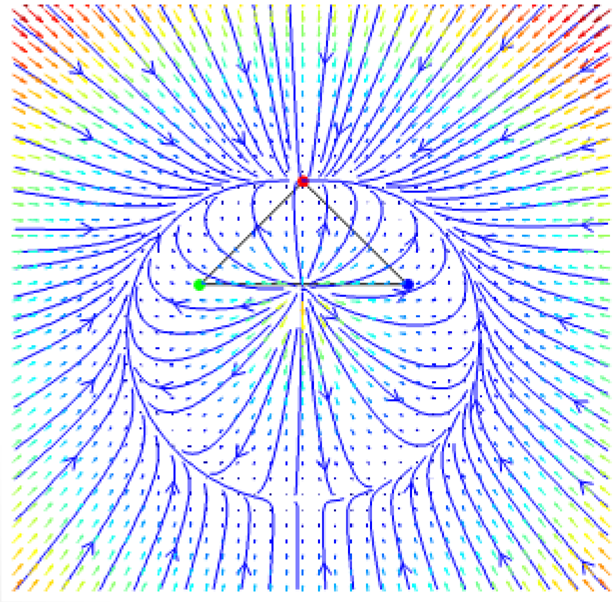
Forces on Inverted Elements

Corotational Force 2D

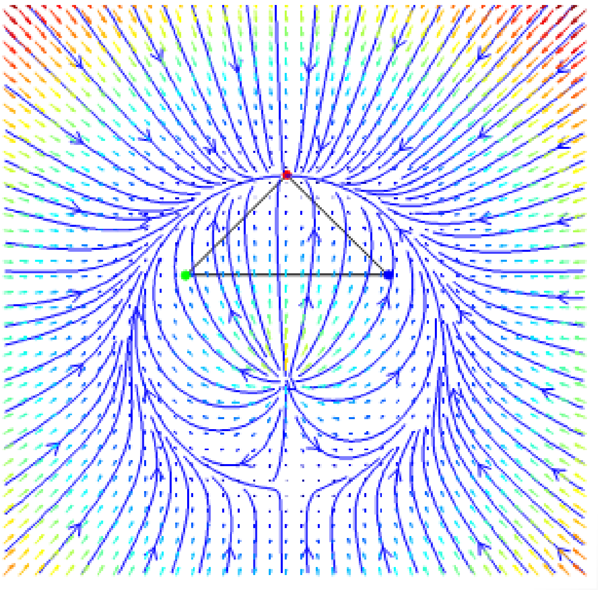
$$\mathbf{f} = \mathbf{RK}(\mathbf{R}^{\mathbf{T}}\mathbf{x} - \mathbf{p})$$



QR



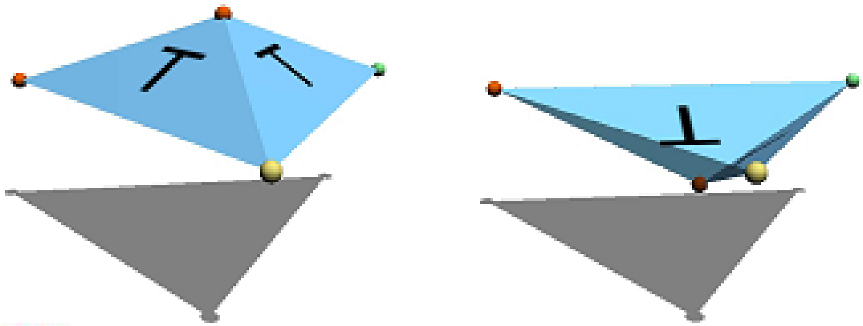
SVD



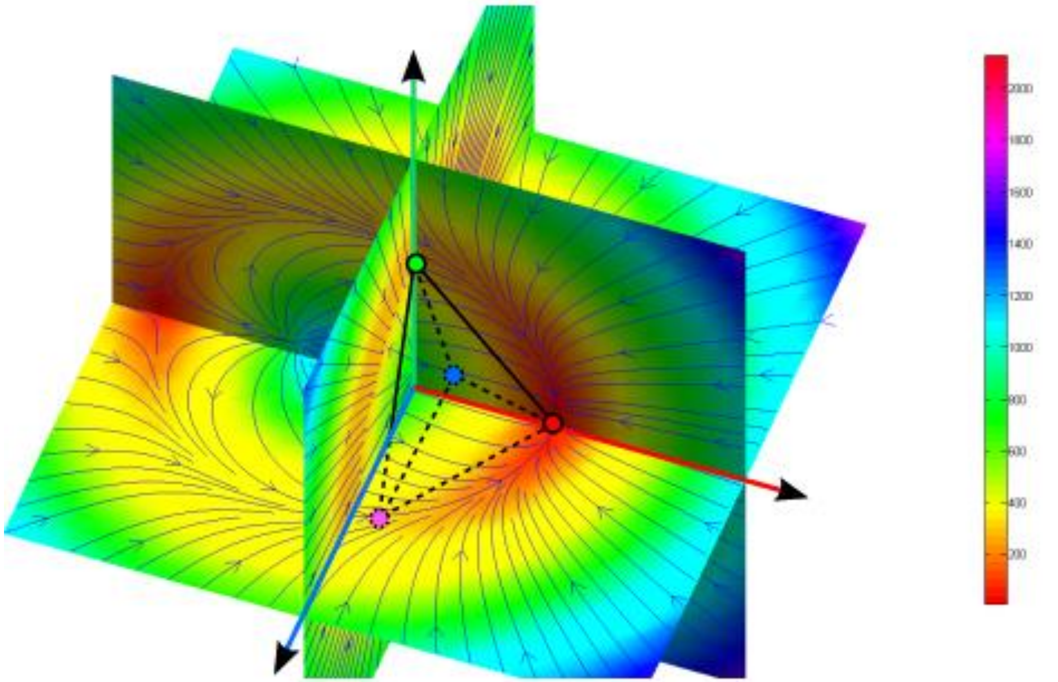
Forces on Inverted Elements

Corotational Force 3D

$$\mathbf{f} = \mathbf{R}\mathbf{K}(\mathbf{R}^{\mathrm{T}}\mathbf{x} - \mathbf{p})$$



SVD



Robust Time-consistent Solution

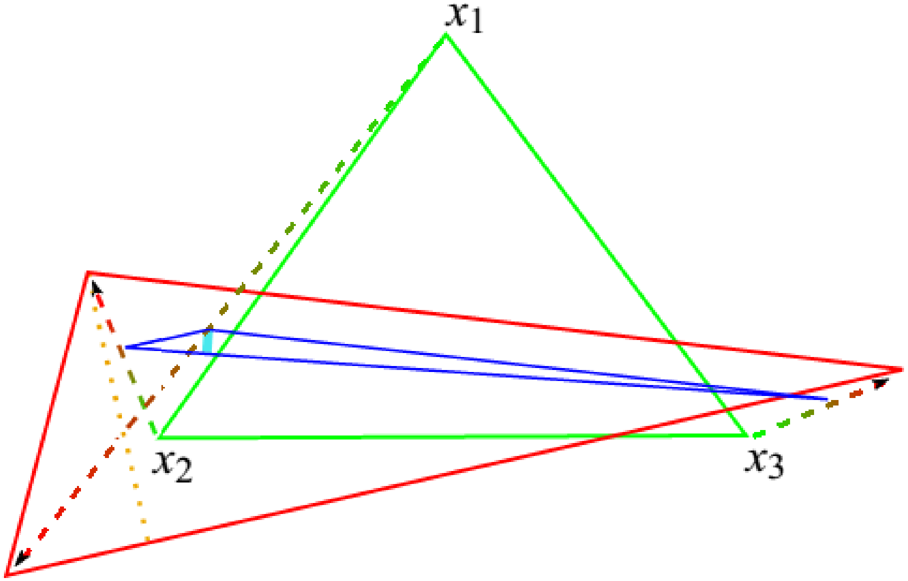
Two main sources of problems

- **Existence of critical points in the computation of** ***R***

Robust Time-consistent Solution

Two main sources of problems

- **Existence of critical points in the computation of *R***
- **Discrete-time heuristic degeneration direction** (Final shortest distance is not enough)

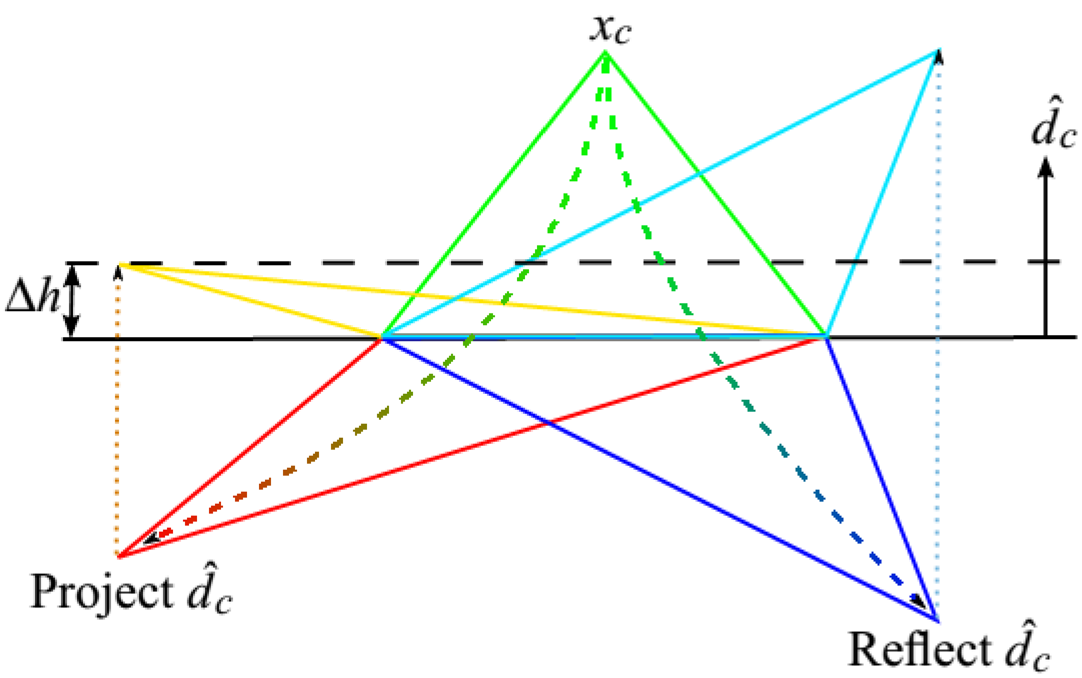


Robust Time-consistent Solution

- **Recovering Direction**: $\hat{d}_c$ normal to the collapse edge.
  - First compute time to collapse t_c
  - Use this to identify proper collapse configuration and define a time-consistent degeneration direction $\hat{d}_c$

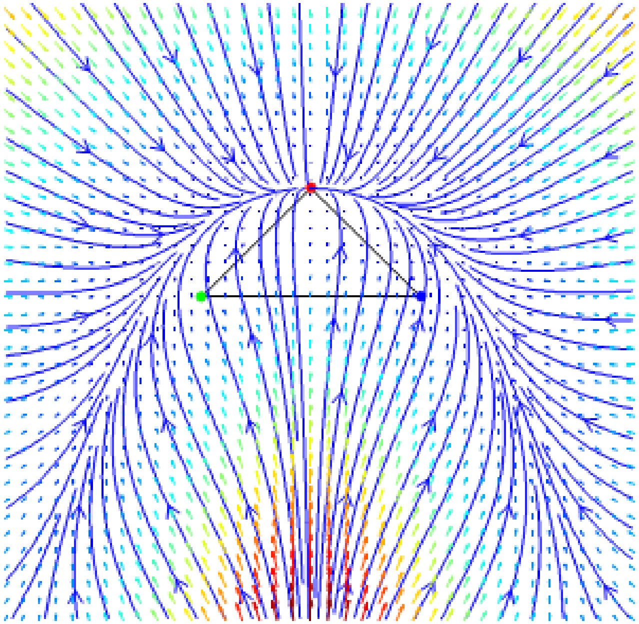
Robust Time-consistent Solution

- **<u>Projection</u> or <u>Reflection</u>** solution: Rotation from a non-degenerate configuration

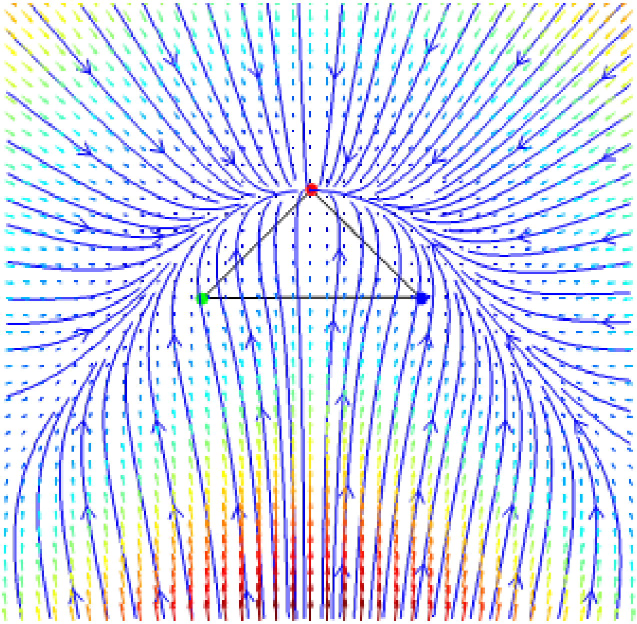


Robust Time-consistent Solution

- **<u>Projection</u> or <u>Reflection</u>** solution: Rotation from a non-degenerate configuration



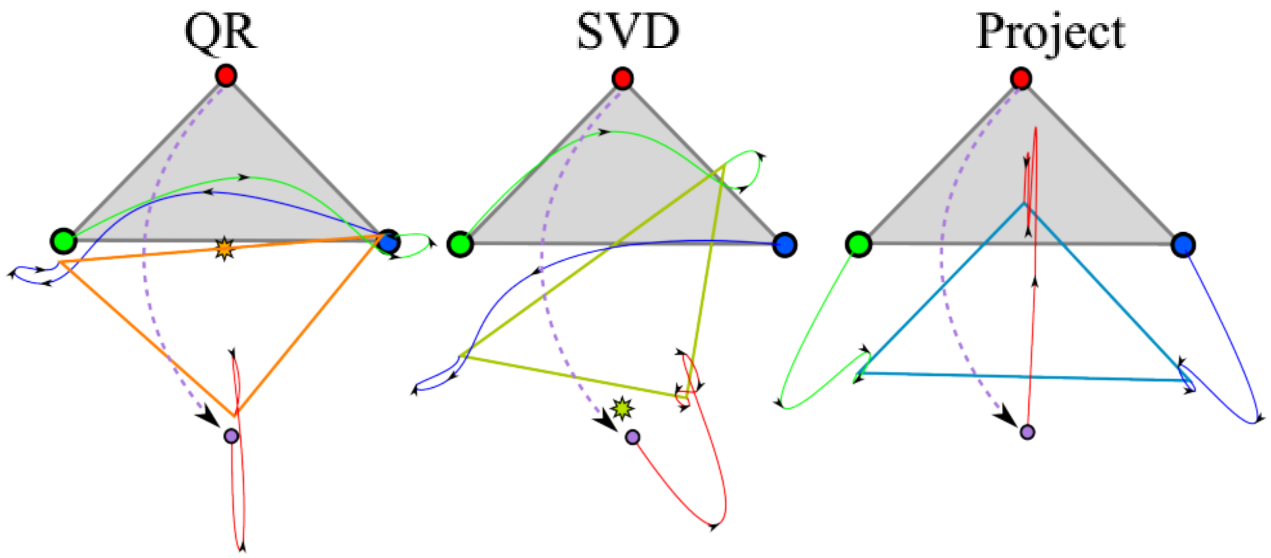
Projection



Reflection

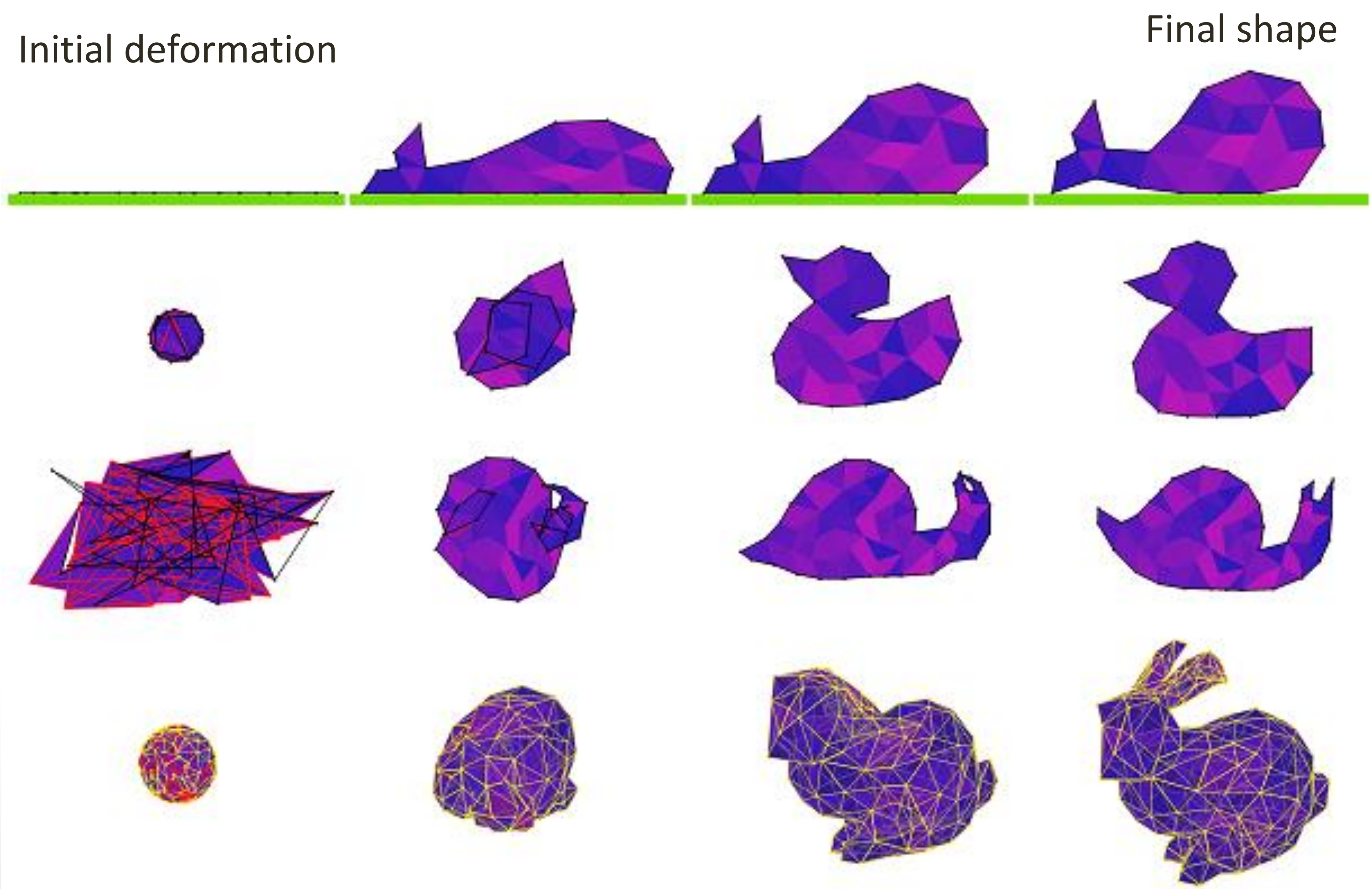
Results:

- Recovering Trajectories



Results:

- Degenerate examples:



Results:

- Examples

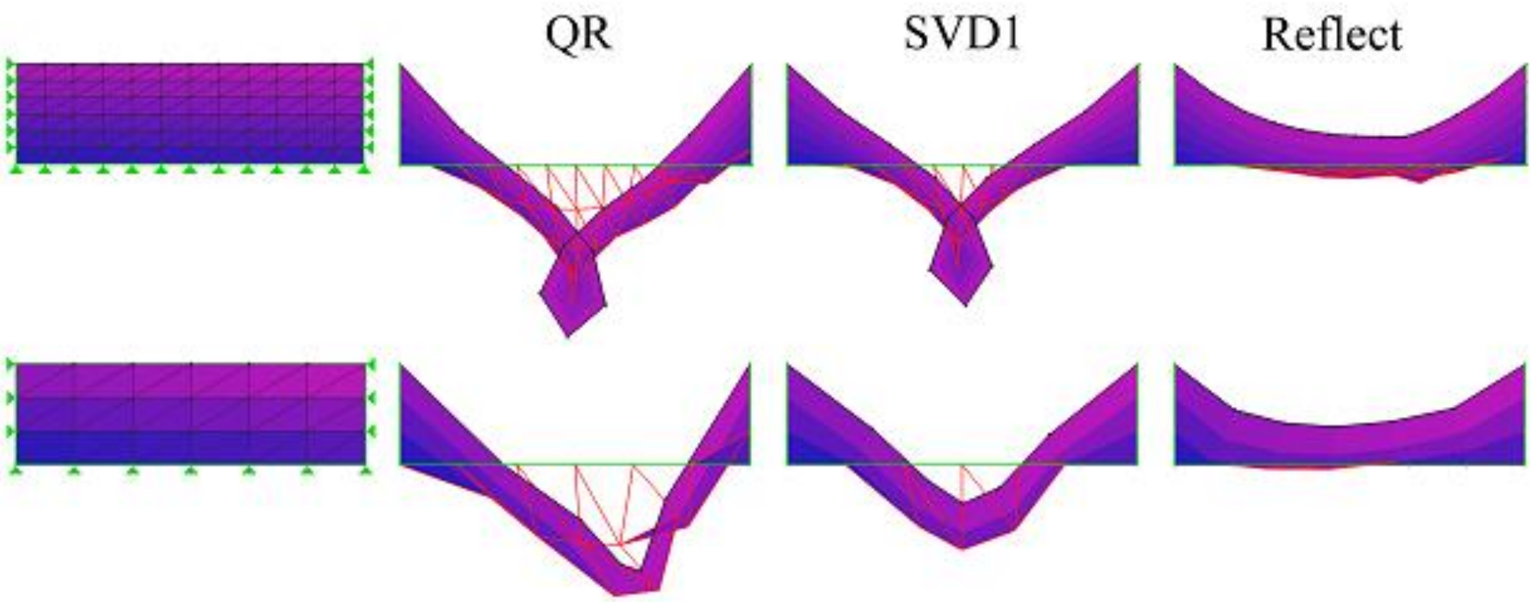
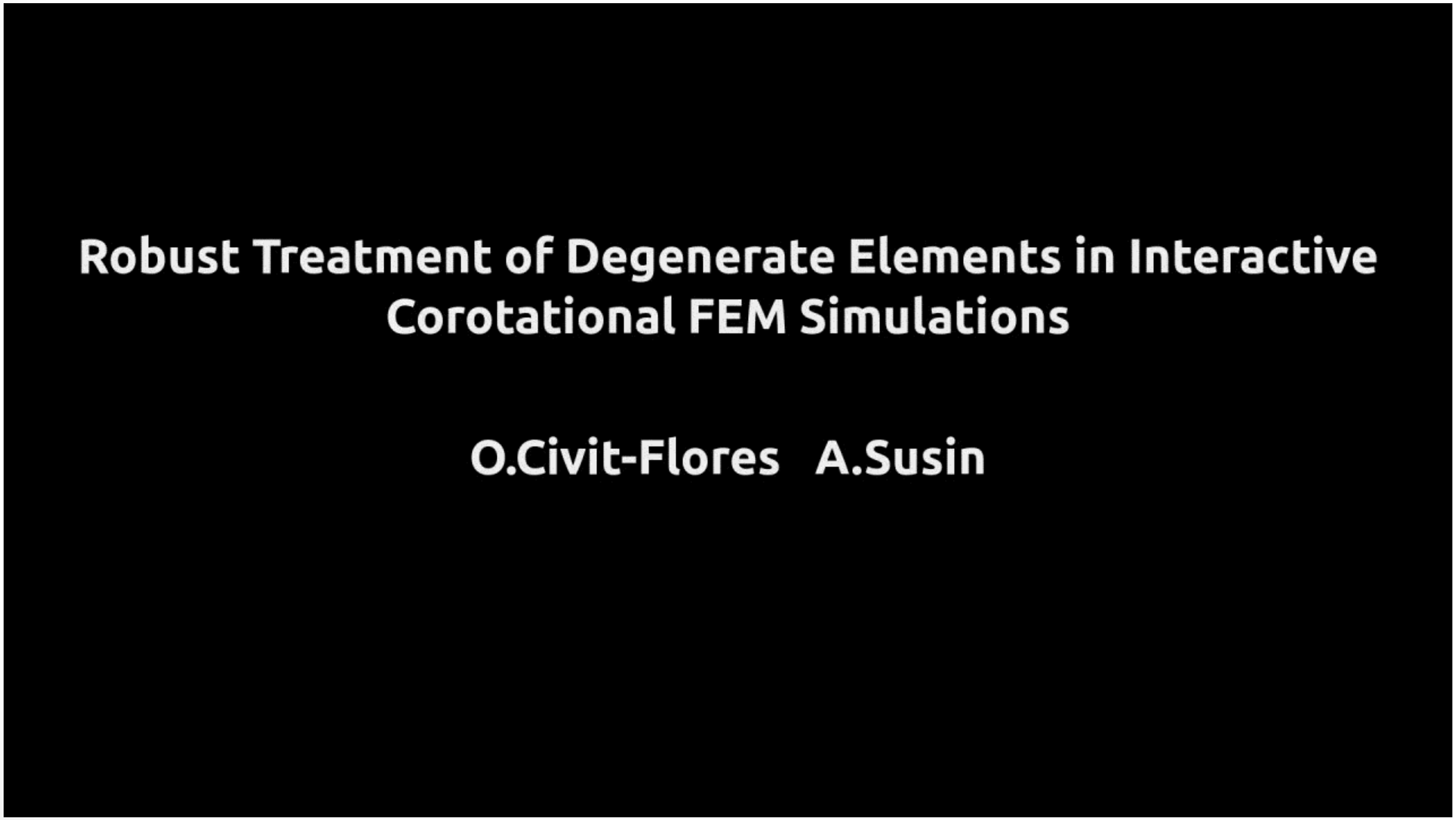
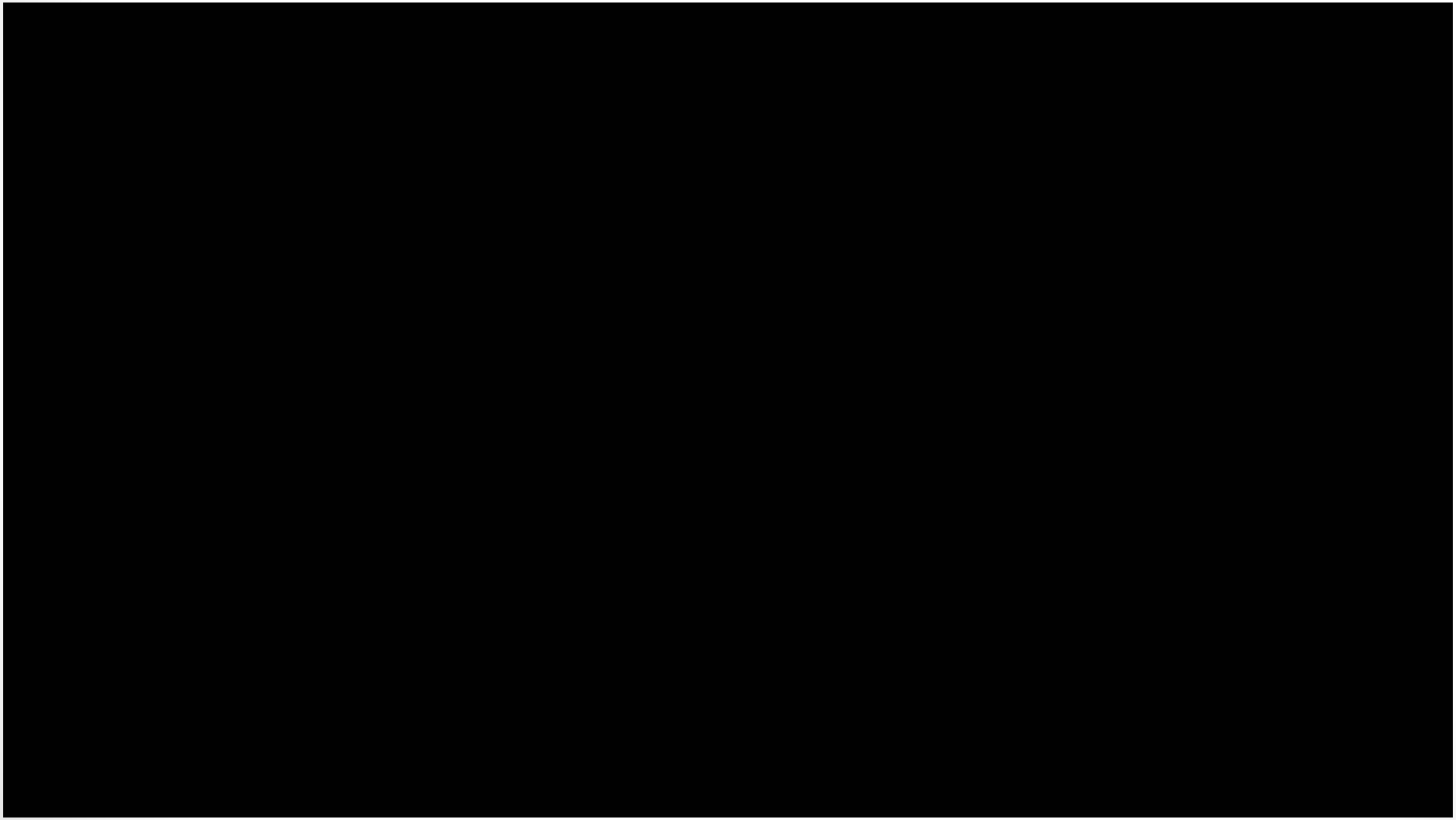


Figure 12: *A supported beam collapses under gravity. Support nodes and edges shown in green, degenerate triangles shown in red wireframe.*

Results:


Robust Treatment of Degenerate Elements in Interactive
Corotational FEM Simulations
O.Civit-Flores A.Susin

Results:



Thanks!!

The problem of inverted elements when simulating deformable objects

Toni Susín

O. Civit-Flores, A. Susín. *Robust Treatment of Degenerate Elements in Interactive Corotational FEM Simulations.* Computer Graphics Forum, Vol 33 (6), pp 298-309 (2014).

Results:

	#Deg.Elem.	QR	SVD1	SVD2	Project	Reflect
Box32	16	0.72	0.27	0.37	0.23	0.26
Box50	24	0.64	0.30	0.21	0.19	0.20
Box128	60	0.64	0.22	0.18	0.17	0.12
Whale31	16	0.25	0.21	0.12	0.09	0.09
Duck43	22	0.29	0.48	0.46	0.36	0.28
Snail87	45	0.23	0.11	0.18	0.12	0.09
Cube384	190	1.06	0.33	0.52	0.30	0.29
Cube1296	630	0.67	0.31	0.28	0.30	0.21
Bunny1085	537	0.60	0.37	0.32	0.26	0.22

Table 1: *Several objects are randomly distorted and their recovery time shown in seconds.*

Results:

	QR	SVD1	SVD2	Project	Reflect
Undeg.2D	14(4%)	55(14%)	55(14%)	31(8%)	31(8%)
Deg.2D	14(4%)	62(15%)	77(18%)	60(15%)	59(15%)
Undeg.3D	77(4%)	918(31%)	918(31%)	212(10%)	212(10%)
Deg.3D	77(4%)	955(32%)	1055(34%)	340(15%)	245(11%)

Table 3: *We simulated a single element during* 10^6 *explicit timesteps in 2D and 3D. For each dimension, in the first simulation the element never degenerates, in the second one the element spents roughly 50% of the timesteps inverted. For each method, we show the average CPU time spent on a single rotation extraction (in nanoseconds) and the percentage of the total computation time it represents.*