On the central configurations of the N–body problem

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Barcelona, Octubre 4, 2017

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The N–body problem consists in study the motion of N pointlike masses, interacting among themselves through no other forces than their mutual gravitational attraction according to Newton's gravitational law.

Equations of motion

The equations of motion are

$$m_k \mathbf{r}''_k = \sum_{j=1, j\neq i}^{n} \frac{G m_j m_k}{r_{jk}^3} (\mathbf{r}_j - \mathbf{r}_k),$$

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r_{jk} is the euclidean distance between m_j and m_k.

Inertial barycentric system

The center of mass of the system satisfies

$$\frac{\sum_{k=1}^{N} m_k \mathbf{r}_k}{m_1 + \ldots + m_N} = \mathbf{a}t + \mathbf{b},$$

where **a** and **b** are constant vectors.

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A such inertial system is called inertial barycentric system. In the rest of the talk we will work in an inertial barycentric system.

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Two configurations are similar if we can pass from one to the other doing a dilatation and/or a rotation.

Homographic solutions of the 3–body problem

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In 1772 Lagrange found two additional homographic solutions in the 3–body problem. Now, the configuration formed by the 3 bodies is an equilateral triangle.

Central configurations

At a given instant $t = t_0$ the configuration of the N–bodies is central if the gravitational acceleration $\mathbf{r}''_k$ acting on every mass point m_k is proportional with the same constant of proportionality to its position $\mathbf{r}_k$ (referred to an inertial barycentric system); i.e.

$$\mathbf{r}''_k = \sum_{j=1, j\neq k}^{n} \frac{Gm_j}{r_{jk}^3}(\mathbf{r}_j - \mathbf{r}_k) = \lambda \mathbf{r}_k, \qquad \text{for } k = 1, \ldots, N.$$

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For spatial central configurations all the homographic solutions only have a dilation.

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(3) If the N bodies are going simultaneously at infinity in parabolic motion (i.e. the radial velocity of each particle tends to zero as the particle tends to infinity), then the particles tend to a central configuration.

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Classes of central configurations

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In what follows we will talk about the classes of central configurations defined by this equivalent relation.

Collinear central configurations

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F.R. MOULTON, The straight line solutions of n bodies, Ann. of Math. **12** (1910), 1–17.

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M. Hampton and R. Moeckel, Finiteness of relative equilibria of the four–body problem, Invent. Math. **163** (2006), 289–312.

A. Albouy and V. Kaloshin, Finiteness of central configurations of five bodies in the plane, Ann. of Math. (2) **176** (2012), 535–588.

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or central configurations of the coorbital satellite problem.

The $(1 + n)$–body problem was first considered by

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In the $(1 + n)$–body problem the infinitesimal particles interact between them under the gravitational forces, but they do not perturb the largest mass.

Let $\mathbf{r}(\varepsilon) = (\mathbf{r}_0(\varepsilon), \mathbf{r}_1(\varepsilon), \ldots, \mathbf{r}_n(\varepsilon))$ be a planar central configuration of the N–body problem with $N = 1 + n$, associated to the masses $m_0 = 1$ and $m_1 = \ldots = m_n = \varepsilon$, which depend continuously on ε when $\varepsilon \to 0$.

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We say that $\mathbf{r} = (\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_n)$ with $\mathbf{r}_i \neq \mathbf{r}_j$ if $i \neq j$ and $i, j \geq 1$, is a central configuration of the planar $(1 + n)$–body problem if there exists the $\lim_{\varepsilon \to 0} \mathbf{r}(\varepsilon)$ and this limit is equal to $\mathbf{r}$.

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PROPOSITION: All central configurations of the planar $(1+n)$–body problem lie on a circle $\mathbb{S}^1$ centered at $\mathbf{r}_0$.

This is the reason for which the central configurations of the $(1+n)$–body problem have applications to the dynamics of the coorbital satellite systems.

PROPOSITION: Let $\mathbf{r} = (\mathbf{r}_0, \mathbf{r}_1, \dots, \mathbf{r}_n)$ be a central configuration of the planar $(1+n)$–body problem. Denoting by θ_i the angle defined by the position of $\mathbf{r}_i$ on the circle $\mathbb{S}^1$, we have

$$\sum_{j=1, j\neq k}^{n} \sin(\theta_j - \theta_k) \left[1 - \frac{1}{8|\sin^3(\theta_j - \theta_k)/2|} \right] = 0,$$

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G.R. HALL, Central configurations in the planar $1+n$ body problem, preprint, 1988 (unpublished).

J. CASASAYAS, J. L. AND A. NUNES, Celestial Mechanics and Dynamical Astronomy **60** (1994), 273–288.

Numerical results due to

H. SALO AND C.F. YODER, The dynamics of coorbital satellite systems, Astron. Astrophys. **205** (1988), 309–327.

n	Number of central configurations
2	2
3	3
4	3
5	3
6	3
7	5
8	3
9	1

For the n's of the table they also study the linear stability of the central configurations.

R. MOECKEL, Linear stability of relative equilibria with a dominant mass, J. of Dynamics and Differential Equations **6** (1994), 37–51.

Numerical computations seem to indicate that

(1) for $n \geq 9$ the number of central configurations is 1 (we know numerically that this is the case for $9 \leq n \leq 100$).

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J.M. CORS, J. L. AND M OLLÉ, Central configurations of the planar coorbital satellite problem, Celestial Mechanics and Dynamical Astronomy **89** (2004), 319–342.

Analytical results

(1) The numerical results of the table for $n = 2, 3, 4$, have been proved analytically by

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A. ALBOUY AND YANNING FU, Relative equilibria of four identical satellites, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **465** (2009), 2633–2645.

(2) For $n \geq 2$ it is known that the regular n–gon, having the infinitesimal particles in its vertices and with the large mass in its center, is a central configuration.

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We only need to prove conjecture (2) for $5 \leq n \leq 8$, when conjecture (1) be proved.

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1 General introduction to central configurations

2 Central configurations of the coorbital satellite problem

3 Central configurations of p nested n–gons

4 Central configurations of p nested regular polyhedra

5 Another open problem on central configurations

Double nested central configurations for the planar $2n$–body problem

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$p = 2$ and $n = 2, 3, 4, 5, 6$.

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S. ZHANG AND Q. ZHOU, Periodic solutions for the $2n$–body problems, Proc. Am. Math. Soc. **131** (2002), 2161–2170.

$p = 2$ and $n \geq 2$.

Triple and Quadruple nested central configurations for the planar $3n$– or $4n$–body problem

J. L. AND L.F. MELLO, Triple and Quadruple nested central configurations for the planar n–body problem, Physica D **238** (2009) 563–571.

$p = 3, 4$ and $n \geq 2, 3, 4$.

THEOREM For all $p \geqslant 2$ and $n \geqslant 2$, we prove the existence of central configurations of the pn–body problem where the masses are at the vertices of p nested regular n–gons with a common center. In such configurations all the masses on the same n–gon are equal, but masses on different n–gons could be different.

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M. CORBERA, J. DELGADO AND J. L., On the existence of central configurations of p nested n–gons, Qual. Theory Dyn. Syst. **8** (2009), 255–265.

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Central configurations of nested regular polyhedra for the spatial 2n–body problem

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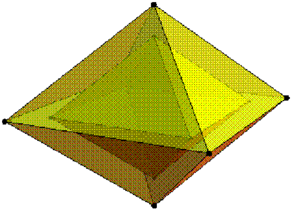
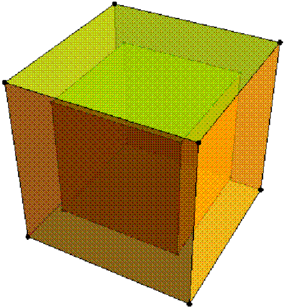
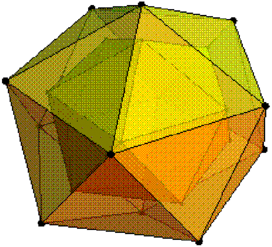


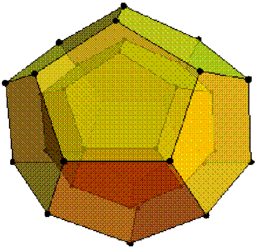
Nested regular octahedra



Nested regular cube



Nested regular icosahedra



Nested regular dodecahedra

Central configurations of 3 nested regular polyhedra for the spatial 3n-body problem

Idem with 3 nested regular polyhedra.

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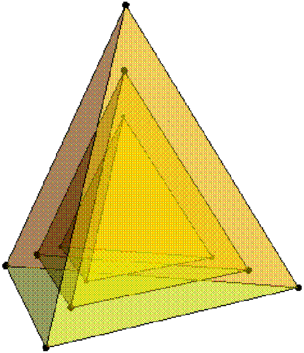
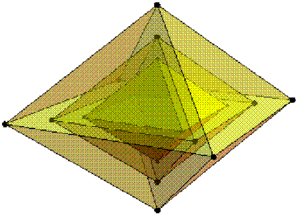
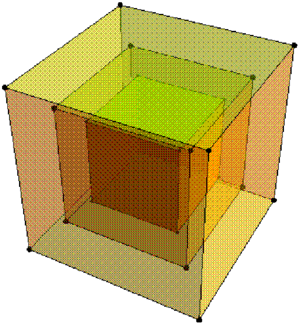


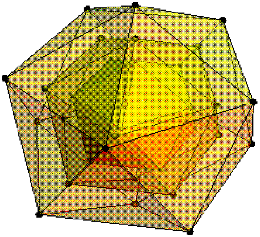
Nested regular tetrahedra



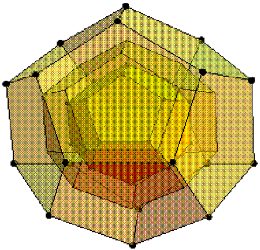
Nested regular octahedra



Nested regular cube



Nested regular icosahedra



Nested regular dodecahedra

On the existence of central configurations of p nested regular polyhedra

THEOREM For all $p \geqslant 2$, we prove the existence of central configurations of the pn–body problem where the masses are located at the vertices of p nested regular polyhedra having the same number of vertices n and a common center. In such configurations all the masses on the same polyhedron are equal, but masses on different polyhedra could be different.

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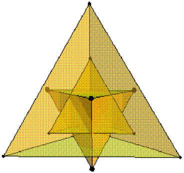
M. CORBERA AND J. L., On the existence of central configurations of p nested regular polyhedra, Celestial Mech. Dynam. Astronom. **106** (2010), 197Ű-207.

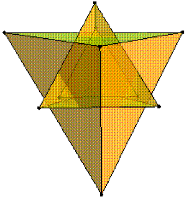
Central configurations of nested rotated regular tetrahedra

Idem but rotated with 3 nested regular polyhedra.

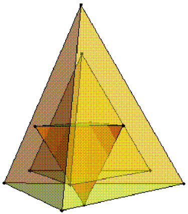
M. CORBERA AND J. L., Central configurations of nested rotated regular tetrahedra, J. of Geometry and Physics **59** (2009), 137–1394.



$$1 \leqslant \rho < R$$



$$1 < R \leqslant \rho$$



$$0 < \rho < 1 < R$$

The contens

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Conjecture 2 has been proved recently in the paper:

A.C. Fernandes, J. L. and L.F. Mello, Convex central configurations of the 4–body problem with two pairs of equal masses, Archive for Rational Mechanics and Analysis **226** (2017), 303–320.

THANK YOU VERY MUCH FOR YOUR ATTENTIONS